Adaptive optics implementation with a Fourier reconstructor

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Adaptive optics takes its servo feedback error cue from the wavefront sensor. The common Hartmann–Shack spot grid, representing the wavefront slopes, is usually analyzed by finding the spot centroids. In a novel application, we used the Fourier decomposition of the spot pattern to find deviations from grid regularity. This decomposition was performed either in the Fourier domain or in the image domain, as a demodulation of the grid of spots. We analyzed the system, built a control loop for it, and tested it thoroughly. This allowed us to close the loop on wavefront errors caused by turbulence in the optical system. © 2007 Optical Society of America

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1. Introduction

Adaptive optics deals with sensing of a distorted wavefront and its correction. There is a variety of applications for adaptive optics, for instance in astronomical observations where we want to discern astronomical objects. The optical signal passes through free space without distortions, but once it encounters the earth's atmosphere the beam is affected and aberrated. There is a need to compensate for the refractive index variation produced by thermal air streams that make the atmosphere turbulent.1 Adaptive optics is also being used for military and ophthalmic applications.2

Wavefront correction is achieved by applying a series of calculated reconstruction commands on the phase modulators. The phase modulator is a device that can modify the wavefront of an incoming distorted beam by adding a refractive or reflective delay, in our case using a membrane deformable mirror. The force on the membrane is affected by a number of parallel actuating capacitors to which voltage is applied. The data for the reconstruction is based on a Hartmann–Shack wavefront sensor which is essentially a lenslet array, sampling the wavefront slope. This correction process must be achieved within a few milliseconds in order to improve the image quality before the aberrations change significantly. In the future, with the advent of sensitive and low-noise detectors, with the development of larger aperture mirror telescopes with thousands of actuators, large deformable secondary mirrors, and huge telescope structures we need to develop faster real-time compensating algorithms to meet these demands, and more efficient schemes for phase compensation must be built. This work is intended as a step in this direction.

 Currently adaptive optics systems use centroid method reconstructors to convert gradient measurements to wavefront phase estimates, where local slopes move beams from their nominal positions and correction is made according to this movement.3 Most works have utilized a least-squares control algorithm and predetection compensation to minimize the wavefront error in an optical system.4–7 The purpose of this work was to test for what is believed to be the first time the implementation of two alternative approaches: Fourier demodulation8 and direct demodulation of the spot pattern9 in order to find the wavefront slope.10 This is multiplied, as in the conventional least-squares approach, by a reconstruction matrix to produce a vector of mirror commands.

In general, the reconstruction process involves two consecutive stages: (1) calculation of the wavefront slopes using the Hartmann spots, or the wavefront curvature from the intensity transport measurement, and (2) calculation of the wavefront itself or the mirror positions, if the deformable mirror is of the piston type. For other phase modulators, such as the bimorph mirror, stage (2) provides the mirror com-
mands. Until now, Fourier analysis was only applied in the second stage for integration of the slopes\textsuperscript{11,12} whereas now we test its real-time employment in the first stage as well. A full Fourier approach is also possible for well-behaved wavefronts where both stages are performed in the Fourier domain: The Hartmann data are transformed, the wavefront gradient isolated and integrated, and the phase is obtained by an inverse transform. This approach was applied for wavefront calculation\textsuperscript{13} but still has to be tested for adaptive optics. Its obvious advantage is the absence of the laborious multiplication by the reconstructor matrix, prohibitively large for extended systems. Since our mirror has a polar and not a Cartesian symmetry, we did not employ this option.

We start our description with the introduction and investigation of the techniques we used for the control scheme and also define a discrete formalism for the deformable mirror in the linear-discrete regime. Next we turn to the optical setup and the other main topic of this paper, detailing the sensing analysis methods that we applied.

2. Control Scheme

Our adaptive optical system is basically a laser beam, which gets distorted in the optical system, reflects off a deformable mirror corrector, and is measured in a wavefront sensor. Our goal here is to flatten the wavefront according to a premeasured reference. The wavefront is measured through an image processing algorithm described below. The control of the wavefront is thus actually performed in terms of its corresponding slopes of the reference wavefront. The corrections of the wavefront are achieved by the use of a deformable mirror, the shape of which gets distorted in the optical system, reflects off the deformable mirror corrector, and is measured in the linear-discrete regime. The most important property of feedback is its ability to cope with modeling errors, external disturbances, and uncertainties.

In our system, the feedback control can be implemented by assigning the input voltage to the actuators on the basis of the mismatch between the actual and desired wavefronts, \( w \) and \( r \). An important point here is that we cannot generate any wavefront as we have only 37 actuators to control 50 variables (an underactuated system). We therefore can track references from the 37-dimensional image space of \( H \) only. This can be interpreted as imposing the constraint \( r = Hr_a \) on the reference signal \( r \), where \( r_a \) is arbitrary. To simplify the design of the feedback controller, or more precisely, to decouple the control loop, we premultiply the tracking error, \( r - w \), by the pseudoinverse of \( H \) (Ref. 14) \[ H^\dagger = (H^H)^{-1}H^H, \]

Thus the data from the Hartmann–Shack sensor are multiplied by \( H^\dagger \), compared with the chosen reference \( r_a \), and this difference is processed by a digital controller \( C_1(z) \) acting at the sampling rate of 30 Hz, as dictated by the video rate (Fig. 2). Signal \( d \) then reflects external disturbances and unmodeled effects. Signal \( n \) reflects the measurements noise, which is brought about by the measurement device, namely, the noise and the background signal in the CCD sensor. Our goal here is to design \( C_1(z) \), which stabilizes the closed-loop system and makes the tracking error \( e = r_a - H^\dagger w \) small despite the presence of \( d \) (which is typically relatively slow) and \( n \) (which is typically relatively fast).

A. Calibration (Identification of \( H \))

The system in Fig. 2 is not yet determined as we do not know the relation between the control variable \( u \) and the processed wavefront \( w \), the wavefront slopes or their Fourier low frequencies. Thus the first step of
the proposed scheme is to identify this relation, i.e., the operator \( H \).

The operator \( H \) is constructed in the calibration process. We identify it by giving the maximal voltage to each separate actuator, one at a time, and the wavefront of each of these pokes is measured by calculating the Fourier demodulation of the spot pattern. The maximal voltage for each poke is chosen to reduce errors attributable to the presence of the measurement noise. Each step of this procedure immediately yields the column of \( H \) corresponding to the exciting actuator. Notice that we simply use Fourier modes, namely the Fourier components of the gradient of the wavefront, in contrast with other processes employing gradients of Zernike modes, and in other cases atmospheric modes.

B. Controller Design

It is readily verified that the closed-loop tracking error \( e = r_a - y \) satisfies

\[
E(z) = S(z)(R_a(z) - D(z)) + T(z)H^aN(z),
\]

where \( S(z) = [I + C_1(z)]^{-1} \) and \( T(z) = [1 + C_1(z)]^{-1}C_1(z) = I - S(z) \) are the sensitivity and complementary sensitivity transfer matrices, respectively. In the choice of the controller \( C_1(z) \) the following requirements were taken into account:

- Simplicity: Since the feedback control law is to be implemented in real time, we cannot afford complicated (i.e., high-order) controllers, the implementation of which would be numerically demanding.
- Set-point tracking: Since the reference vector \( r_a \) is assumed constant, we require a zero steady state error, \( \lim_{t \to \infty} e(t) = 0 \), for all constant \( r_a \) and \( d_z \). This amounts to requiring that the static gain of the sensitivity transfer matrix be zero, \( S(1) = 0 \), or, equivalently, that the static gain of the controller, \( C_1(1) \), be infinite.
- Low sensitivity to measurement noise: The measurement noise \( n_z \) is typically fast, so to prevent its amplification by the feedback loop we require that the magnitude of the complementary sensitivity frequency response, \( T(e^{j\omega}), \theta \in [-\pi, \pi] \), at high frequencies be low (smaller than 1). This requirement can be cast as \( |T(-1)| < \alpha \), for some \( \alpha \leq 1 \), i.e., as a constraint on \( |T(z)| \) at the highest frequency.

The first two requirements suggest the following digital decentralized proportional-integral (PI) form of the controller:

\[
C_1(z) = z^{-1}k_p\left(1 + \frac{k_az}{z - 1}\right)I,
\]

where \( k_a \) is the proportional gain, and \( k_p \) is the accumulator (the presence of the integrator term guarantees that the static gain of this controller is infinite). Note that \( C_1(z) \) has a pole at the origin to reflect inevitable computational delay.

Some lengthy, albeit straightforward, algebra yields then that the third requirements, combined with the standard closed-loop stability requirement, imposes the following constraints on the controller parameters:

\[
\begin{align*}
  k_p &> -1, \\
  k_p k_a &> 0, \\
  k_p(2 + k_a) &< \frac{2\alpha}{1 + \alpha}, \\
  k_p(2 + k_a) &> -\frac{2\alpha}{1 - \alpha}, \text{ if } \alpha < \frac{1}{2}.
\end{align*}
\]

The gray areas in Fig. 3 show admissible pairs \((k_p, k_a)\) for several values of \( \alpha \).

Note that the stability conditions are actually recovered as \( \alpha \to \infty \). As \( \alpha \) decreases, the admissible areas shrink. The requirement of a small high-frequency gain is especially restrictive in the first quadrant, where the proportional gain cannot be increased much. Since a low proportional gain gives rise to slow transient response, this essentially implies that negative controller coefficients are preferable. Indeed, in the third quadrant we can still use a high proportional gain, even if \( \alpha \) is small.

3. Experiment

To test the system and control, we built a simple optical system (Fig. 4). The measurement was performed by using a lenslet array, and compensation was produced by a membrane mirror (OKO Technologies 37-channel electrostatic mirror). This was a part from a package from Adaptive Optics Associates, Inc. (AOA), which also included a Tk/Tcl language program for Hartmann–Shack wavefront sensing and for adaptive optics closed-loop control and diagnostic graphics. We first blocked the deformable mirror arm and took a reference image for calibration of the wavefront sensor itself. Then we blocked the reference arm and measured the aberrations in the
other arm. To calibrate the system, we poked each mirror element in turn and measured the wavefront. Alternatively, we poked a whole mode (a combination of elements) to find its response. We inverted the responses by using singular value decomposition, to construct a matrix response to any given wavefront through application of voltage to the relevant actuators. After testing the system by correcting for static errors, we moved on to dynamic correction: A hair dryer was operated at lower voltage to create weak turbulence across the optical path in front of the deformable mirror. In all cases full correction was achieved.

A. Fourier Reconstruction
The common method today for constructing the wavefront phase distortion is by sampling its local slopes using a lenslet array and applying a digital closed-loop process. The slopes are measured at the focal plane of the lenslet array as foci shifts. A centroiding algorithm finds the location of each focal spot, from which the whole map of slopes is constructed, to serve as inputs for the control loop.

To calculate the local shifts of the foci we also used two new methods: First, we performed a Fourier demodulation of the Hartmann–Shack pattern. This method extracts the essential data of the wavefront behavior, namely the shifts of focal spots, by considering them as modulation of the full regular spot pattern. Its disadvantage is the requirement for a full Fourier transform at all possible frequencies, even irrelevant ones. Second, we conducted a direct demodulation of the foci frequency only, without reverting to Fourier transforms. This helped us to reduce the processing times.

For the first method, we used a 2D fast Fourier transform (FFT) to obtain the slope of the wavefront phase. That slope appears in the complex side lobes of the transform, where these side lobes correspond to the lenslet period. One can try to retrieve it from the real, imaginary, amplitude, phase, or hermitian parts of the transform. While the hermitian option is theoretically the best, it requires an integer frequency of spots, which was not possible for us, or Fourier plane interpolation, not available through the given software. Instead, we chose experimentally the most suitable part of the transform by realizing that it should have the highest rms response to the same set of input wavefront aberrations. The 200-odd Hartmann–Shack spots were imaged with a CCD camera holding 480 × 640 pixels. To avoid aliasing we removed the side columns and added empty rows to create a 512 × 512 pixel array. Not surprisingly, the amplitude part of the transform yielded the highest response to the aberrations. Unfortunately, the loss of phase part of the transform inhibited their use, and indeed an adaptive optics control loop became unstable. Instead, we chose the imaginary part of the transform, which proved to be much more reliable and stable.

To build the reconstruction matrix, we tried a number of schemes, and settled on a poke reconstructor: The response of the wavefront to a constant voltage on each actuator resulted in a different transform pattern. We took the imaginary part of the two side lobes corresponding to the x and y slopes in the transform and stacked them one on top of the other for display purposes (Fig. 5), or staggered them into a linear array as a column input. Each poked actuator gave such a column, and their combination provided the response of the system. That response matrix has the width of the number of actuators (37), and the height of the combined number of pixels in the two side lobes (50). Each such matrix element contained the imaginary part of the Fourier component of the wavefront slope. From it was subtracted the reference response matrix without any voltage applied to the actuators (the calibration frame). The arrangement of the data in the slope vectors is not important but should be kept consistent between calibration

![Fig. 4. The optical system. A collimated, wide laser beam enters from the bottom right and splits (in beam splitter BS2) into either a reference channel or a deformable mirror channel. Light from the unblocked arm returns into the wavefront sensor (WFS: Hartmann–Shack lenslet array and camera). An image of the corrected laser beam is split (in beam splitter BS1) into another camera. Turbulence is added between the lenses in front of the deformable mirror.](image1)

Fig. 5. (Color online) Selected part of the Fourier transform of the current measurement of the wavefront (less a reference array). Only the imaginary part of two lobes is shown, corresponding to the x and the y slopes. On top is a square of 25 pixels of the 0, 1 lobe, below it the square of the 1, 0 lobe.

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and operation. Next we used a singular value decomposition (SVD) algorithm to determine the pseudoinverse of the response matrix. Thresholding removed the weaker eigenvalues of the inverse. Thus the procedure to construct a reconstructor matrix is the same as for centroiding, except that the Fourier modes of the slopes are used and not the slopes themselves.

In running the adaptive optics loop, each Hartmann–Shack frame is Fourier transformed, and the same vector is formed from its two orthogonal side lobes (again, the imaginary values of 25 elements around the main frequency in each direction). This vector is now multiplied by the reconstructor matrix to yield the next mirror commands for the controller.

B. Smoothing Reconstruction

Time is a critical factor in a fast-changing system, and thus a minimum of CPU cycles is required to close the adaptive optics loop. Even though the Fourier transform is very efficient, we have just seen that we may use a very small number of Fourier components out of the many calculated. For example, we were able to close the loop with 25 x-lobe and 25 y-lobe Fourier components, out of the $512^2 = 262,144$ in the full frame. It makes more sense to transform only the necessary Fourier components, and this can be achieved by phase detection: Since we know the frequency $k_0$ of the Hartmann spots, we multiply by $e^{ik_0x}$, and integrate by smoothing the complex result. Smoothing can be achieved by convolution with a kernel the width of the Hartmann pitch, which is too slow for a real-time process. Instead, we shifted and added the complex array to half its values, and then again at half the pitch. This is equivalent to finding only the $k_0$ harmonic in one direction. In a temporal sense this is similar to superheterodyne or locked-in detection. Finally we extracted the phase of the smoothed array. This phase is the $x$ component of the slope. The process was repeated for the $y$ direction. The results are shown in Fig. 6.

The analysis is performed in the image domain and hence is very fast. As the resulting number of points (slope components) is equal to the number of camera pixels inside the aperture, much larger than the number of actuators, coarser smoothing is possible. Following the smoothing and phase extraction, we simply sampled regularly the aperture for the slope (once for each direction) in $2 \times 25$ points as an input for the control loop. From this point on we proceeded exactly as in the centroiding scheme: The phases from these 50 samples made a single slope vector, used to create the response matrix for each of the 37 actuators. SVD created the reconstructor matrix. Then each new Hartmann–Shack pattern was twice multiplied, smoothed, and down sampled to obtain the same slopes vector, to be used in the mirror control loop.
1. Timing and Control

Using the Tk/Tcl and C programs provided by AOA we were able to close the control loop in 80 ms (30 Hz frame rate) in the classical centroid approach. In the next stage we tried the Fourier and smoothing phase extraction methods. The inefficiency of the FFT algorithm and the shear frame size made the process much slower, closing the loop in approximately 600 ms. At this rate we were still able to correct aberrations due to slow turbulence (Fig. 7). No attempt was made to subsample or bin down the frame, or locate a more efficient FFT or similar algorithm. The reason is that the AOA software package is no longer supported, and the inclusion of new C subroutines is not possible. As a result, the smoothing algorithm control loop exceeded 2500 ms. Its application in C or MATLAB was, on the other hand, much faster than the previous methods. Using a different computer, it was applied successfully to a living human eye.

We also verified the control schemes for different parameters. By changing the hair dryer speed we induced weak and strong turbulence changes, which translated into corresponding temporal and spatial aberration scales. Indeed we found that our model led to areas of divergence and uncontrolled oscillations and areas of stable operation, exactly matching our model. We tested many points in the \( (k_p, k_a) \) plane, and we show one example in Fig. 8. Future work, on a more advanced system, will test our theory on the propagation of noise and further validate the control model.

4. Summary

We operated what we believe to be the first adaptive optics control system utilizing Fourier phase extraction. In one realization we used a full Fourier transform of the Hartmann spots, and extricated the phases (namely the wavefront slopes) from the orthogonal side lobes as control inputs. We were able to close the adaptive optics loop in acceptable time, which can be further reduced by improved FFT algorithms or hardware. Fourier modes might be better descriptors of turbulence-created phase and of deformable mirrors with Cartesian or hexagonally tiled actuators. For very large systems they should be the natural choice. Add to that the advantage that strong aberrations are better tolerated in Fourier analysis, as there is no need to contain the Hartmann–Shack spots within the designated lenslet areas. Thus the lenslets can have longer focal lengths, and increase the phase sensitivity. Yet another advantage lies in taking the first side lobes: High-frequency noise is filtered out, making the use of a hardware aperture redundant.

All these advantages exist also in the second realization, where we demodulated the same Hartmann grid in the image domain, and sampled it inside the deformable mirror. These sample points were also used to close the loop and correct the wavefront for
different turbulence conditions. Our control system functioned as predicted and was able to follow and immediately correct even strong cases of turbulence. In a separate experiment we were able to close the loop on a living eye using the same method.

There is an advantage to sampling the phase of the interpolated wavefront: Sample locations can be judiciously chosen according to criteria other than the given lenslet positions. For example, their density and spread can be guided by the turbulence spectrum and by the wind: Sampling can be denser in the upwind direction to improve prediction. In ocular adaptive optics, spherical aberration can be dominant, which also calls for more sampling around the pupil periphery. Other factors can be the aperture shape or boundary and the positions of the actuators. It might be advantageous to sample where the slopes of the influence functions of the actuators are maximal and the sensitivity is better. Sampling can also be optimized experimentally as in the SVD process. We intend to further investigate these points since it might be possible to minimize or even avoid the matrix multiplication in the control loop altogether if the right sampling choice is taken.

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References