1 SO(4) symmetry of triplet superconductivity and antiferromagnetism in Bechgaard salts

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Recent experiments give compelling evidence that superconductivity in Bechgaard salts involves spin-triplet electron pairing. The proximity of triplet superconductivity and antiferromagnetism in these compounds prompts us to study the interplay between the two orders in quasi-one dimensional electron systems. We find that the two orders are unified in a natural way through an SO(4) symmetry group, and we show that SO(4) is a good approximate symmetry of the system near the phase transition between the two orders, without the need for fine-tuning of microscopic parameters. We study the experimental consequences of SO(4) symmetry, including predictions for the phase diagram and for the low-energy excitation spectrum of Bechgaard salts.

1.1 Introduction. Competing orders in strongly correlated electron systems

A common feature among many strongly correlated systems is the close proximity, or even the coexistence, of multiple ordered states.

1.2 Emergence of higher symmetries

Symmetry is a powerful principle in elucidating the properties of a complex system. The symmetry group of a quantum mechanical Hamiltonian contains information regarding the degeneracy of states, and provides a scheme by which to organize the spectrum of excitations. In situations where the symmetry of the Hamiltonian is spontaneously broken with the onset of long range order, the pattern of symmetry breaking determines the spectrum of gapless Goldstone modes uniquely [4].

Although the symmetry of the Hamiltonian is explicit in most cases, there are situations in strongly correlated electron systems where, for special values of parameters, the Hamiltonian displays a higher symmetry than manifest. This typically occurs at the phase boundary between two seemingly unrelated
orders, where the extra symmetry generators rotate the degenerate order parameters into one another. The addition of new Goldstone modes leads to a suppression of the critical temperature. In addition, the enhanced symmetry group constrains the form of the Ginzburg-Landau free energy for the competing orders, leading to strong constrains on the topology of the phase diagram near the phase transition.

These ideas are illustrated by the Hubbard model on a bipartite lattice \[21, 22\],

\[H = -t \sum_{\langle ij \rangle, \sigma} c_i^\dagger \sigma c_j \sigma + U \sum_i (n_{i\uparrow} - \frac{1}{2})(n_{i\downarrow} - \frac{1}{2}) - \mu \sum_i n_i \sigma.\]

The model is invariant under spin \(S_0(3)\) rotations, \([H, S_\alpha] = 0\), generated by the total spin operators, \(S_\alpha = \frac{1}{2} \sum_c \hat{c}_c^\dagger \sigma_{s\alpha} c_c \sigma\), and under the \(SO(2)\) group of phase rotations, \([H, Q] = 0\), which are generated by the total charge \(Q = \frac{1}{2} \sum c_i \sigma (n_i - \frac{1}{2})\). Exactly at half filling (\(\mu = 0\)), the charge group is enlarged by the inclusion of two new generators, \(\eta^-\) and \(\eta^+ \equiv (\eta^-)^\dagger\), where

\[\eta^- = \sum_i (-1)^i \hat{c}_i \downarrow \hat{c}_i \uparrow,\]

as can be verified through explicit calculation,

\[\{H, \eta^\pm\} = \mp 2\mu \eta^\pm, \quad (1.1)\]

The “pseudospin” operators \(\eta_x = \frac{1}{2}(\eta^+ + \eta^-)\), \(\eta_y = \frac{i}{2}(\eta^+ - \eta^-)\), and \(\eta_z = Q\) satisfy an \(SO(3)\) algebra. The \(\eta\) are spin-singlet operators, and therefore commute with the \(S_\alpha\), \([S^\alpha, \eta^\pm] = 0\). Hence, the total symmetry group is \(SO(4) \approx SO(3)_{\text{spin}} \times SO(3)_{\text{pseudospin}}\).

In the negative-\(U\), electrons in the ground state like to form on-site spin-singlet pairs. These pairs may condense to form a singlet superconductor, or they may instead prefer to arrange themselves into a checkerboard CDW. In fact, these two types of order are connected by the pseudospin group,

\[\{\eta^\alpha, \Delta^b\} = i\epsilon^{abc} \Delta^c, \quad (1.2)\]

where \(\Delta^x = \text{Re}\{\sum_i \hat{c}_i \downarrow \hat{c}_i \uparrow\}\), \(\Delta^y = \text{Im}\{\sum_i \hat{c}_i \uparrow \hat{c}_i \downarrow\}\), and \(\Delta^z = \frac{1}{2} \sum_{i\sigma} (-1)^i n_i \sigma\). Thus, by symmetry, singlet superconductivity (\(\Delta^x, \Delta^y\)) and checkerboard CDW (\(\Delta^z\)) are actually degenerate at half-filling.

Since the two types of order are degenerate, it is possible to construct a new class of low energy states by alternating regions with local SC and CDW orders that slowly twist into one another. For short-ranged interactions, the energy of such states is made arbitrarily small by making the twist arbitrarily slow. Thus, enhanced symmetry leads to gapless Goldstone modes associated with the pseudospin operators, \(\eta\), which generate the rotations between the two orders. In the quantum system, such low energy modes are created by
acting with $\eta$ on the ground state, as seen in equation (1.1). Away from $\mu = 0$, the excited state is a massive pseudo-Goldstone mode whose gap softens as the SO(4) symmetric point $\mu = 0$ is approached to yield a true Goldstone mode.

The dynamics of these low energy modes can be captured by a quantum rotor model [6], which is obtained by coarse-graining the lattice into small clusters, and then projecting onto the low energy Hilbert space on each cluster. By symmetry, the remaining degrees of freedom must decompose into well-defined representations of SO(4). In this case, they are mapped into a local three dimensional vector order parameter $\Delta_i$ on cluster $i$, subject to the local constraint $|\Delta_i|^2 = 1$. The quantum rotor model then consists of a kinetic energy term for each rotor, in addition to a ferromagnetic coupling for rotors on adjacent clusters.

The presence of new low energy modes leads to enhanced fluctuations of the order parameters, and therefore to a reduction in critical temperature $T_c$. For instance, in two dimensions, it is possible to have a finite temperature phase transition into a long-range checkerboard CDW (which is a discrete order parameter), or into a quasi long-range-ordered superconductor. However, the negative-$U$ Hubbard model at half-filling has an SO(3)-symmetric order parameter, which, by the Mermin-Wagner theorem, cannot be spontaneously broken at finite temperature. Hence, enhanced fluctuations suppress $T_c$ all the way to zero in this case.

Symmetry principles have been introduced to study the competition of order parameters in a variety of experimental systems. In S.C. Zhang’s SO(5) theory of high $T_c$ superconductivity [23], antiferromagnetism and $d$-wave superconductivity are treated as components of a five dimensional vector order parameter. In addition to the generators of the usual charge SO(2) and spin SO(3) symmetries, new $\pi$-operators are introduced, which rotate superconductivity and antiferromagnetism into each other. A combination of analytical approximations and numerical results can be used to argue an approximate SO(5) theory of a class of two dimensional lattice models, such as the Hubbard and the $t$-$J$ model [7,12]. The SO(5) symmetry has also been used to discuss quasi two-dimensional organic $\kappa$-BEDT-TTF salts [14]. The unification approach based on higher symmetries has been generalized to several other types of competing states. SO(5) and SO(8) symmetries have been used to classify possible many-body ground states in electronic ladders [10, 18]. SO(6) symmetry has been introduced to discuss competing striped phases and superconductivity in the cuprates [11]. SO(4) symmetry has been used to combine $d$-wave superconductivity and $d$-density wave phases [9,16]. It has also been suggested that the SO(5) algebra can be used to combine ferromagnetism and triplet superconductivity in quasi two-dimensional $\text{Sr}_2\text{RuO}_4$ [15], although the existence of microscopic models with such symmetry has not been demonstrated.
1.3 SO(4) symmetry in quasi-one dimensional systems

1.3.1 SO(4) is a natural symmetry of AF and TSC orders

Here, we will argue that quasi-one dimensional Bechgaard salts near the transition between AF and TSC orders have an enhanced SO(4) symmetry. We can understand the generators of this symmetry group by examining the order parameters that they act upon. For a quasi-one dimensional system, it is natural to assume that the orbital component of the triplet superconducting order parameter is $p_x$, where $x$ is the intrachain direction. Hence, the Néel and triplet superconducting order parameters are

$$N_\alpha = \frac{1}{2} \sum_{k,s,s'} \left( a_{k,s}^\dagger \sigma_{s,s'}^\alpha a_{k,s'} + a_{k,s'}^\dagger \sigma_{s,s}^\alpha a_{k,s} \right)$$

$$\Psi_\alpha^\dagger = \frac{1}{2} \sum_{k,s,s'} a_{k,s}^\dagger (\sigma_2 \sigma_3)^{\alpha,s} a_{k,s'}^\dagger$$

Here $a_{k,s}^\dagger$ creates right/left moving electrons of momentum $\pm k + k_f$ and spin $s$.

The total spin and charge are

$$S_\alpha = \frac{1}{2} \sum_{r,k,s,s'} a_{r,k,s}^\dagger \sigma_{s,s'}^\alpha a_{r,k,s'}$$

$$Q = \frac{1}{2} \sum_{k,s} \left( a_{k,s}^\dagger a_{k,s} + a_{k,s}^\dagger a_{k,s} - 1 \right)$$

The spin operators $S_\alpha$ form an SO(3)$_{\text{spin}}$ algebra of spin rotations, $[S_a, S_b] = i e^{a b c} S_c$, under which $N_\alpha$ and $\Psi_\alpha$ transform as vectors. On the other hand, the charge $Q$ rotates the real and imaginary parts of $\Psi_\alpha$ into one another, while maintaining the real vector $N_\alpha$ invariant. What is missing for a unified description of AF and TSC orders is an operator $\Theta^\dagger$, which rotates $\Psi_\alpha$ into $N_\alpha$. Examination of the order parameters shows that such an operator must have momentum $\pm 2k_f$, charge $\pm 2$, and spin 0,

$$\Theta^\dagger = \sum_k \left( a_{k,\uparrow}^\dagger a_{-k,\uparrow} + a_{k,\downarrow}^\dagger a_{-k,\downarrow} \right)$$

The operators $\Theta$, $\Theta^\dagger$ are combined with the charge to define the “isospin” generators, $I_x = \frac{1}{2}(\Theta^\dagger + \Theta)$, $I_y = \frac{i}{2}(\Theta^\dagger - \Theta)$, $I_z = Q$, which satisfy an isospin SO(3)$_{\text{iso}}$ algebra $[I_a, I_b] = i e^{a b c} I_c$. These spin and isospin algebras are independent of one another, $[I_a, S_\alpha] = 0$, so that the total group defined by these generators is SO(4)$\approx$SO(3)$_{\text{iso}} \times$SO(3)$_{\text{iso}}$. The vectors $\Psi$, $N$ are then combined naturally into a single tensor.
\[ \hat{Q} = \begin{pmatrix} \text{Re } \Psi_x & \text{Im } \Psi_x & N_x \\ \text{Re } \Psi_y & \text{Im } \Psi_y & N_y \\ \text{Re } \Psi_z & \text{Im } \Psi_z & N_z \end{pmatrix} \]  

The rows (columns) of \( \hat{Q} \) transform as a vector under the spin (isospin) SO(3) algebra, \([S_\alpha, Q_{\beta \gamma}] = i \epsilon^{\alpha \beta \gamma} Q_{\delta \gamma}\) (\([I_\alpha, Q_{\beta \gamma}] = i \epsilon^{\alpha \beta \gamma} Q_{\delta \gamma}\)). Hence \( \hat{Q} \) transforms in the (1,1) representation of the SO(4) algebra.

Having defined the group SO(4), we next address its commutation relations with the Hamiltonian. On general grounds, the total charge \( Q \) is a good quantum number of any closed system. Furthermore, for systems in the absence of magnetic fields or spin-orbit coupling, the total spin \( S_\alpha \) also commute with the Hamiltonian. Microwave absorption experiments in (TMTSF)_2AsF_6 measured [20] the anisotropy in the exchange couplings to be \( 10^{-6} \), which is unlikely to play a significant role in the competition between AF and TSC phases. Hence, we conclude that both \( Q \) and \( S_\alpha \) are good symmetries of the Hamiltonian everywhere in the phase diagram. In the following section, we will show that, for Luttinger liquids at incommensurate fillings, the Hamiltonian has a much larger symmetry SO(4)_{iso} \times SO(3)_{spin}. Then, in SECTION XXX we will argue that the effect of umklapp at commensurate fillings is to reduce SO(4)_{iso} to the group SO(3)_{iso} described above. Then, in SECTION XXX we will discuss the experimental consequences of such symmetry.

### 1.3.2 SO(4) \times SO(3) symmetry Luttinger liquids at incommensurate fillings

Unlike half-filling, there is no mechanism to pin a SDW in a clean incommensurate Luttinger liquid. Hence, the SDW order parameter is a complex vector,

\[ \Phi_\alpha = \sum_{k,k'} a_{+,k}^\dagger \sigma_\alpha^a a_{-,k'}, \]

The phase diagram of interacting electrons in one dimension was obtained in Ref. [2?] using bosonization and renormalization group analyses. This system has a phase boundary between SDW and TSC phases when the Luttinger parameter in the charge sector, \( K_\rho \), equals 1. As shown in Ref. XXX, the Hamiltonian of a Luttinger liquid at \( K_\rho = 1 \) and incommensurate filling has an SO(4)_{iso} \times SO(3)_{spin} symmetry. This is generated by the operators \( S_\alpha, Q, \Theta, \) and \( \Theta^\dagger \) of the previous section, and the new operators \( \Delta Q, A, \) and \( A^\dagger \), defined by

\[ \Delta Q = \frac{1}{2} \sum_{k,s} \left( a_{+,k}^\dagger a_{+,k} a_{-,k} - a_{-,k}^\dagger a_{-,k} a_{+,k} \right). \]
\[ A = \sum_k \left( a_{+,k}^\dagger a_{+,k}^\dagger + a_{-,k}^\dagger a_{-,k}^\dagger \right). \]
$\Delta Q$ is the difference in charge between right and left movers. This is a symmetry of the low energy Hamiltonian, which does not contain umklapp scattering at incommensurate filling.

While we will not show here that the charge sector has an SO(4)$_{iso}$ symmetry explicitly, such a large isospin symmetry group at incommensurate filling at $K_{\rho o} = 1$ can be understood by analogy with the spin sector. In the absence of backward scattering ($g_1 = 0$), the Luttinger parameter in the spin sector $K_{\sigma}$ equals 1, and the spin Hamiltonian does not have a sine-Gordon term. Thus, the spin Hamiltonian for $g_1 = 0$ has the same form as the charge Hamiltonian at the TSC/AF boundary. Furthermore, we know that, when $g_1 = 0$, the spin of right and left movers is conserved separately, so that the full spin-symmetry in this case is SO(3)$\times$SO(3)$\approx$SO(4)$_{spin}$. Thus, we conclude that the charge sector at $K_{\rho} = 1$ and in the absence of umklapp must have an SO(4)$_{iso}$ symmetry. In fact, the generators $\Theta$ and $\eta$ can be obtained by bosonizing the spin operators of right and left movers, converting spin variables into charge variables, and refermionizing.

1.3.3 SO(4) symmetry of quasi-one dimensional systems at half-filling

1.3.4 SO(4) symmetry in a strongly anisotropic Fermi liquid

1.4 Competition of spin density wave order and triplet superconductivity in Bechgaard salts.

Real materials are only quasi one-dimensional. Coupling between chains gives rise to finite temperature phase transitions and is likely to break the exact microscopic symmetry at the phase boundary. However, as long as 3D coupling is much weaker than the intrachain tunnelling and interactions, we expect to find approximate microscopic symmetry.

To study the phase diagram of systems with competing AF and TSC orders we consider a Ginzburg-Landau (GL) free energy

$$F = \frac{1}{2} (\nabla Q_{a\alpha})^2 + r Q_{a\alpha}^2 + \delta r (Q_{z,a}^2 - Q_{x,a}^2 - Q_{y,a}^2)$$
$$+ \tilde{u}_1 Q_{a\alpha}^2 Q_{b\beta}^2 + \tilde{u}_2 Q_{a\alpha} Q_{a\beta} Q_{b\alpha} Q_{b\beta}$$

(1.8)

We assume that temperature and pressure control the quadratic coefficients $r$ and $\delta r$. When $\delta r = 0$ the model has full SO(4) symmetry. Away from this line it only has spin and charge SO(3)$\times$SO(2) symmetry. The quartic terms in (1.8) are the only ones consistent with SO(4) symmetry between AF and TSC. We also note that if one derives the GL energy for weakly interacting quasi one-dimensional electrons following the usual approach, one obtains the model in (1.8) with $\tilde{u}_1 = 21\zeta(3)/16\pi^2 v_f T^2$ and $\tilde{u}_2 = -7\zeta(3)/8\pi^2 v_f T^2$ [?].
The properties of model (1.8) depend strongly on the sign of $\tilde{u}_2$, which determines whether the triplet superconductor is unitary ($\Re \psi \propto \Im \psi$) or non-unitary ($\Re \psi \times \Im \psi \neq 0$). We expect the unitary case, $\tilde{u}_2 < 0$, to be of experimental relevance to (TMTSF)$_2$PF$_6$, and in the remainder of this paper we concentrate exclusively on this case. The mean field diagram is then composed of an AF phase separated from a TSC phase by a first order phase transition, and a disordered (Normal) phase separated from the two other phases by second order lines, see Fig. 1.1.

\[
\begin{array}{c}
\text{AF} \\
\downarrow \\
\text{normal} \\
\downarrow \\
\text{TSC}
\end{array}
\]

Fig. 1.1. Mean field phase diagram of eq. (1.8) in the unitary case $\tilde{u}_2 < 0$. There is a first order transition (thick line) between AF and TSC phases.

To understand the role of thermal fluctuations in model (1.8), and in slightly perturbed models where the quartic coefficients do not lie exactly on the SO(4) symmetric manifold, we use $4 - \epsilon$ renormalization group (RG) analysis. We find that the RG equations have only two fixed points: a trivial Gaussian fixed point $\bar{\delta}r = \delta r_1 = \delta r_2 = 0$ and an SO(9) Heisenberg point $\bar{\delta}r \neq 0, \delta r_1 = 0, \delta r_2 = 0, \tilde{u}_2 = 0$. All RG flows starting with $\tilde{u}_2 \neq 0$ are runaway flows. This analysis can be generalized to order parameters $\mathbf{N}$ and $\mathbf{\Psi}$ that are $N$-component vectors, in which case the SO(4)$\approx$SO(3)$\times$SO(3) symmetry becomes SO(3)$\times$SO($N$). However, find that even in the large $N$ limit, all flows with $\tilde{u}_2 < 0$ are runaway flows, indicating the absence of fixed points with unitary TSC.

The absence of a fixed point in the RG flow often implies that fluctuations induce a first order phase transition, thus precluding a multicritical point in the phase diagram. In order to inspect this possibility, we study model (1.8) directly in $d = 3$ dimensions in the large $N$ limit. The results are shown in the Inset of Figure 1.2. We find a first order transition between AF and TSC phases along the SO(4) symmetric line $\delta r = 0$, as predicted by mean field theory. However, the large $N$ results differ from the mean field theory in two important aspects: the transition between normal and TSC phases is first order and, close to the SO(4) symmetric line, fluctuations induce a first order transition between AF and TSC phases.
The first order Normal/TSC transition was proposed in the context of $^3$He by Bailin et al. [1]. It remains a theoretically open question whether this transition is first order for $N = 3$. However, for large $N$ the Normal/TSC transition remains first order arbitrarily far from the SO(4) symmetric point $\delta r = 0$, and is therefore not a consequence of SO(4) symmetry. On the other hand, the AF/TSC transition is only first order close to the SO(4) symmetric line, and is a direct consequence of the enhanced fluctuations due to SO(4) symmetry.

If we assume that the experimentally controlled pressure changes an extensive variable conjugate to $\delta r$, such as the volume of the system, the first order transition broadens into a coexistence region of TSC and AF. This is consistent with the experimental phase diagram for (TMTSF)$_2$PF$_6$ shown in Fig. 1.2.

![Fig. 1.2.](image)

**Fig. 1.2.** (a) Phase diagram of model (1.8) in the large $N$ limit. Thick lines represent first order phases. The direct transition between AF and TSC phases, tuned by $\delta r$, is first order even at mean field level for $N = 3$. Enhanced fluctuations due to SO(4) symmetry make the AF/Normal transition first order near the AF/TSC phase boundary. (b) Schematic temperature-pressure phase diagram of (TMTSF)$_2$PF$_6$. N/AF and AF/TSC correspond to coexistence regimes of the appropriate phases. CITE CITE

1.5 Experimental tests of the SO(4) symmetry in Bechgaard salts.

We now proceed to discuss possible experimental tests of SO(4) symmetry. To study the spectrum of low energy collective excitations in the neighborhood of the SO(4) symmetric point we propose a quantum rotor model,

$$\mathcal{H}_t = \frac{1}{2\chi_1} \sum_i S_i^2 + \frac{1}{2\chi_2} \sum_i I_i^2 - J \sum_{(ij)\alpha\alpha} Q_{i,\alpha\alpha} Q_{j,\alpha\alpha}$$
In this lattice model each site has spin and isospin vector operators $S_i$ and $I_i$, and an $SO(4)$ tensor order parameter $Q_{i,a \alpha}$. They satisfy the commutation relations 
\[ [S_i, \alpha, S_j, \beta] = i \delta_{ij} \epsilon_{\alpha \beta \gamma \delta} S_i, \gamma, [I_i, a, I_j, b] = i \delta_{ij} \epsilon_{abc} I_i, c, [S_i, \alpha, Q_j, a \beta] = i \delta_{ij} \epsilon_{\alpha \beta \gamma \delta} Q_j, \gamma \alpha, [I_i, a, Q_j, b \alpha] = i \delta_{ij} \epsilon_{abc} Q_j, c \alpha, \] 
and $[Q_i, a \alpha, Q_j, b \beta] = 0$. In equation (1.9) the unit length constraint of the rigid rotors is replaced by interaction terms $\tilde{u}_1$ and $\tilde{u}_2$. We obtained the collective mode spectrum by writing Heisenberg equations of motion for order parameters and symmetry generators, linearizing these equations in the ordered states, and solving the ensuing eigenvalue problem. In addition to the Goldstone (massless) modes due to the exact $SO(3) \times SO(2)$ symmetry, there are pseudo-Goldstone (massive) modes due to the approximate $SO(4)$ symmetry. The latter soften as we approach the $SO(4)$ symmetric point $\delta r = 0$.

![AF and TSC phases](image)

**Fig. 1.3.** Low energy collective excitations in AF and unitary TSC phases for short range interactions. In the AF phase, there is a doublet of spin waves $S$ and a doublet of massive isospin waves $I$. Due to translational symmetry breaking, the $2k_f$ modes also appear near $k = 0$ (dashed lines). In the TSC phase, there is a phase mode $\phi$, a doublet of spin waves $S$, and a massive $\Theta$ mode. The massive modes in both phases soften as the phase transition is approached.

The symmetry breaking pattern in each ordered phase determines the low energy spectrum, see Fig. 1.3. In the AF phase there is a (degenerate) doublet of massless spin waves, and a doublet of massive isospin waves. Their dispersion about $k = 2k_f$ is $\omega_{AF, S}(k) = N \sqrt{J/\chi_2} |\Delta k|$ and $\omega_{AF, I}(k) = N \sqrt{J/\chi_2} + J/\chi_2 (\Delta k)^2$, where $\Delta k = k - 2k_f$. Of these, only the spin waves couple to neutron scattering. In the TSC phase we find a doublet of massless spin waves, a massless phase mode $\phi$, and a massive $\Theta$ mode. In the presence of long range Coulomb interactions, the $\phi$ mode is shifted to the plasma frequency. The spin modes are centered at $k = 0$, ...
\[ \omega_{TSC,S}(k) = \psi \sqrt{\frac{J}{\Delta \chi_1}} |k|, \text{ whereas the } \Theta \text{ mode has a minimum at } k = 2k_f, \]

\[ \omega_{TSC,\Theta}(k) = \psi \sqrt{\frac{4\pi}{\Delta \chi_2} + \frac{J}{\Delta \chi_2}} (\Delta k)^2. \]

At low temperatures the intensity of spin polarized neutron scattering is proportional to the dynamic spin structure factor \( \chi_{zz}(q,\omega) = \sum_n |\langle n| S_q^z |0 \rangle|^2 \delta(\omega - \omega_{n0}) \). Here \( |0\rangle \) is a ground state and \( n \) summation goes over all excited states. The \( \Theta \) operator creates an eigenstate of the system \( |\Theta\rangle = \frac{1}{N} \Theta |0\rangle \). The contribution of \( |\Theta\rangle \) to the spin structure factor at \( Q = (2k_f, 0, 0) \) is

\[
\chi_{\Theta}(Q, \omega) \propto |\langle 0|\Psi^2|0 \rangle|^2 \delta(\omega - \omega_{\Theta}).
\] (1.10)

Hence, we observe that the \( \Theta \) excitation appears as a resonance in inelastic neutron scattering [8,13,17] and its intensity is proportional to the square of the pairing amplitude.

From equation (1.5) we observe that the \( \Theta \) excitation is a collective mode in the particle-particle channel (i.e. it has charge \( Q = 2 \)). Deep in the normal phase it cannot be probed by conventional methods, such as electromagnetic waves or neutron scattering, as these only couple to particle-hole channels (e.g. spin or density). The situation changes when the system becomes superconducting. In the presence of a condensate of Cooper pairs charge is not a good quantum number and particle-particle and particle-hole channels mix. This makes the \( \Theta \) excitation accessible to neutron scattering experiments in the TSC phase. For quasi one dimensional \( (\text{TMTSF})_2\text{PF}_6 \) we expect strong pairing fluctuations even above \( T_c \). Hence, precursors of the \( \Theta \) resonance should be visible in the normal state, with strong enhancement of the resonant scattering intensity appearing when long range TSC order develops [5].

The most important feature of the \( \Theta \)-resonance, which identifies it as a generator of the SO(4) symmetry, is the pressure dependence of the resonance energy inside the TSC phase. When the pressure is reduced and the system is brought toward the phase boundary with the AF phase, we predict the energy of the \( \Theta \)-resonance to be dramatically decreased. Mode softening is not expected generically at first order phase transitions and provides a unique signature of the SO(4) quantum symmetry.

We note that due to interchain hopping, the center of mass momentum of the \( \Theta \) excitation in quasi-one dimensional systems is \( (2k_f, \pi, \pi) \).

Another approach to detect the \( \Theta \) excitation involves tunneling experiments with the SSC/(TMTSF)$_2$ClO$_4$ junction shown in Fig. 1.4 (analogous experiments in the context of \( \pi \) excitations in the high \( T_c \) cuprates are discussed in Ref. [2]). A singlet superconductor provides a reservoir of Cooper pairs that can couple to \( \Theta \) pairs in \((\text{TMTSF})_2\text{ClO}_4\). One needs to overcome, however, the momentum mismatch between the two types of pairs. A possible approach is to use an intermediate layer of the quasi 1d material \((\text{TMTTF})_2\text{PF}_6\). This salt is quarter filled and displays spin-Peierls (SP) order. The modulations of the SP order thus have a periodicity of four TMTTF sites, matching the \( (2k_f, \pi, \pi) \) wave vector of \((\text{TMTSF})_2\text{ClO}_4\). The small mismatch between the two wave vectors, due to differences in the lattice constant
in these compounds, can be compensated by a parallel magnetic field [19]. We expect peaks in the current-voltage characteristics of the junction when the voltage bias compensates the energy difference between Cooper and $\Theta$ pairs

$$2eV = \omega_\Theta$$ (1.11)

Peaks in $IV$ should be present even above the superconducting transition temperature of (TMTSF)$_2$ClO$_4$ and only require the other material to be superconducting. The choice of (TMTSF)$_2$ClO$_4$ is made as this material is likely to be close to the AF/TSC transition at ambient pressure [3]. This eliminates the need for pressure cells, which would make the experiments much more difficult.

<table>
<thead>
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<th>SC</th>
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<td>(TMTTF)$_2$PF$_6$</td>
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Fig. 1.4. Tunneling experiment for detecting the $\Theta$ excitation in (TMTSF)$_2$ClO$_4$ material. A singlet superconducting material with a higher transition temperature than (TMTSF)$_2$ClO$_6$ provides a reservoir of Cooper pairs that can couple resonantly to $\Theta$ pairs. Momentum mismatch between the Cooper pairs in SC and $\Theta$ pairs in (TMTSF)$_2$ClO$_4$ is compensated by scattering of electrons in a layer of the SP material (TMTTF)$_2$PF$_6$.

References


