Flux and Charge Controlled Cooper Pair Pumping

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1. Charge pumps
2. Pumped charge and Berry phase in Josephson junction circuits
3. Flux and charge controlled pump - sluice
4. Experiments

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Normal state charge pumps: Principle

Cyclic operation of gates, $q_i = C_{gi} V_{gi}/e$, transports charge unidirectionally, even in the absence of bias.

H. Pothier et al. 1992
Currents on the level of few pA
Accuracy about 1% with respect to $I = e f$
Charge pumps

Practical whys:
Towards current standard

Metrological "Quantum Triangle"

Normal single-electron pump: \( I = e f \)


High accuracy but slow: \( I < 10 \) pA
Why to use Josephson junctions instead?

1. Cooper pair pumps are much faster than normal electron pumps:

   $$I_{\text{MAX}} = I_C \times \text{duty cycle}$$

   Adiabatic behaviour up to

   $$j_{LZ} \sim \frac{E_J^2}{\hbar E_C}$$

2. Possibility to study geometric effects

3. Possibility to employ combined gate (charge) and flux control – in normal pumps typically only charge control possible

$I_C$ controlled by magnetic flux
Basics of Josephson junctions

Josephson relations:

\[ I_S = I_C \sin \varphi \]
\[ \hbar \frac{d\varphi}{dt} = 2eV \]
\[ \varphi = \phi_1 - \phi_2 \]

Important energy scales of a Josephson junction with critical current \( I_C \) and capacitance \( C \):
- Josephson coupling energy
  \[ E_J = \frac{\hbar}{2e} I_C \]
- Charging energy
  \[ E_C = \frac{(2e)^2}{2C} \]
- Thermal energy
  \[ k_B T \]
"Classical" Josephson junctions

RCSJ-model

Ideal Josephson junction

\[ I \]

\[ V \]

\[ C \]

\[ R \]
Josepshon junctions in the charging limit ($E_C >> k_B T$)

$E = E_C (Q_0/e - n)^2$

- $n$ even

$E = E_C (Q_0/e - n)^2 + \Delta$

- $n$ odd

$Q_0 \equiv C_g V_g$

Normal state, $\Delta = 0$

Superconducting state, $\Delta > E_C$

M. Tuominen et al. (1992)
Charge and phase

\[ [\varphi, Q] = i2e \]

**Classical Josephson junctions:**
\[ E_J \gg E_C : \varphi \text{ fixed, } Q \text{ completely undetermined} \]

**Coulomb blockaded Josephson junctions:**
\[ E_J \ll E_C : \varphi \text{ completely undetermined, } Q \text{ fixed} \]

Problem to generate large currents accurately
The first three-junction CPP


Potential sources of error in Cooper pair pumping

1. higher order processes due to finite $E_J$
2. non-adiabaticity
3. environmental impedance
4. background charge noise
5. quasiparticles
R-pumps

Suppression of higher order tunneling effects by dissipative environment

S. V. Lotkhov, a) S. A. Bogoslovsky, b) A. B. Zorin, and J. Niemeyer
Physikalisch-Technische Bundesanstalt, D-38116 Braunschweig, Germany
APL 2001
Transported charge by supercurrent and pumping

\[ \langle \hat{I}_\ell \rangle = \langle m | \hat{I}_\ell | m \rangle + 2 \Re \text{e} \langle m | \hat{I}_\ell | \hat{m} \rangle \]

usual supercurrent

\[ \langle m | \hat{I}_\ell | m \rangle \equiv I_{S,\ell} = \frac{\partial E_m}{\partial \varphi} \]

geometric contribution due to non-stationary control parameters

\[ Q_{\text{cycle}} \equiv \oint \langle \hat{I}_\ell \rangle dt = \oint I_{S,\ell} dt + Q_P \]

\[ Q_P = 2\hbar \sum m \oint \sum_{n \neq m} \frac{\langle m | \hat{I}_\ell | n \rangle}{E_m - E_n} \langle n | \nabla \tilde{q} m \rangle \cdot d\tilde{q} \]

Pumped charge and Berry phase

\[ \theta_{\text{Berry}} = i \oint \langle m | dm \rangle \quad \leftrightarrow \quad Q_P = 2 \Re \int \langle m | \hat{Q}_e | dm \rangle \]

\[ Q_P = -2e \frac{\partial \theta_{\text{Berry}}}{\partial \varphi} \]


A pumping experiment does not require strict phase bias, measurement of Berry phase however does.
Three-junction Cooper pair pump - revisit

Perfectly phase-biased adiabatic CPP

\[ Q_P = 2\hbar 3m \left[ \sum_{n \neq m} \int \frac{\langle \hat{I}_t \rangle_{mn}}{E_m - E_n} \langle n|\partial_{\tilde{q}m}\rangle \cdot d\tilde{q} \right] \]

\[ Q_P / (2e) \approx 1 - 9E_J / E_C \cos \varphi \]

At \( \varphi = 0 \) this gives all the current; at general bias \( Q_P \) adds to the average of direct supercurrent \( I_S \propto E_J \)

Charge and Flux controlled pump - sluice

Tunable SSET: $E_{J1}(\Phi_1), E_{J2}(\Phi_2), q$ – two valves and one piston

Temporal suppression of $E_J$ allows for fast operation with small errors

Charge and Flux controlled pump - sluice

Illustrative analysis in two-state approximation: $E_{J,\text{max}} \ll E_C$

$|g\rangle = e^{i\gamma}|a\rangle|0\rangle + |b\rangle|1\rangle$, $|e\rangle = e^{i\gamma}|b\rangle|0\rangle - |a\rangle|1\rangle$

$\gamma$ determined by fluxes only:

$\gamma = \arctan\left(\frac{E_{J2} - E_{J1}}{E_{J1} + E_{J2}} \tan \frac{\phi}{2}\right)$

Amplitudes determined by charge and fluxes:

$|a|^2 = 1 - |b|^2 = \frac{1}{2} \left[ 1 - \frac{q - 1/2}{\sqrt{(q - 1/2)^2 + (E_{12}/E_C)^2}} \right]$  

$E_{12} = \frac{1}{2} \sqrt{E_{J1}^2 + E_{J2}^2 + 2E_{J1}E_{J2}\cos \phi}$
Obtaining pumped charge and Berry phase in two-state approximation

Direct evaluation of charge as

\[ Q_P = 2\Re \int \langle m | \hat{Q}_\ell | dm \rangle \]

involves contributions that arise from different sections in the cycle than those of Berry phase

\[ \theta_{\text{Berry}} = - \oint |a|^2 d\gamma \]

\[ \gamma = \arctan \left( \frac{E_{J2} - E_{J1}}{E_{J1} + E_{J2}} \tan \frac{\varphi}{2} \right) \]
Berry phase and $Q_p$ in a sluice

In a two state approximation, Berry phase yields using:

$$Q_p = -2e \frac{\partial \theta_{\text{Berry}}}{\partial \varphi}$$

$$\frac{Q_p}{(2e)} = 1 - 2\frac{\sqrt{E_j^2 + E_C^2}}{E_j E_C} E_{J,\text{res}} \cos \varphi$$

Both Berry phase and direct integration of charge yield:

$$\frac{Q_p}{(2e)} \approx 1 - 2\frac{E_{J,\text{res}}}{E_j} \cos \varphi$$

in the small $E_j/E_C$ limit.
Finite frequency errors of the device

Note: Several pairs per cycle can be pumped

To study the non-adiabaticity errors, these results were obtained by solving Schrödinger equation and integrating in time, not by adiabatic approximation.
First experiments

Set-up:

Device:

SQUID loops

Input coils

Junctions

Gate line

\( f = 0 - 20 \text{ MHz} \)
Experimental gate and flux modulation

\[ \hat{H} = \frac{2e^2}{2C_J + C_g} (\hat{n} - n_g)^2 - E_J \left( \pi \frac{\Phi_r}{\Phi_0} \right) \cos(\phi + \varphi/2) \]
\[ - E_J \left( \pi \frac{\Phi_l}{\Phi_0} \right) \cos(\varphi/2 - \phi). \]
General $I/V$ curves, pumping

3 MHz, 4...34 pairs / cycle pumped
Quantitative comparison to $I = N2ef$

Improvements?

1. Better temporal suppression of $E_J$ using a SQUID array or a three-junction SQUID?

2. Higher speed via
   (a) increasing $E_J$ (by lowering junction resistance or ultimately by using Nb junctions)
   (b) pulse optimisation to avoid non-adiabaticity
Second generation samples
Characteristics of the improved samples
Nanoampere pumping

Optimum operation point at finite bias voltage: no phase-bias, incoherent tunneling(?)

Maximum current pumped is about 5% of $I_C$: Theoretical limit is about 15% based on the pulsing cycle employed.

Counting of charges in a cycle possible from the step structure of the current vs gate amplitude.
Measurement scheme of Berry-phase in a sluice

The sluice-pump acts as a current source, which induces an additional $\pm Q_P f$ on the "big" Josephson junction threshold detector.

$$I \pm Q_P f = I_{BIAS}$$

$$\theta_{Berry} = \frac{1}{2e} \int^{\varphi} Q_P (\varphi') d\varphi'$$

In a separate threshold experiment with a comparable Josephson junction we have demonstrated sub-1 nA resolution of switching asymmetry (cond-mat/0612087), which should be sufficient for the BP measurement.
Controlled transport in small Josephson junction networks provides an interesting system to study quantum pumping effects and geometric phases. A measurement of Berry phase in a Josephson junction array seems feasible.

Flux and charge controlled Cooper pair pump can present a fast and accurate choice as a current source in metrological applications. Only three control parameters are necessary. Recent experiments have demonstrated the operation principle and high current yield.