Classical versus quantum transport near quantum Hall transitions

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Transport data near quantum Hall transitions are interpreted by identifying two distinct conduction regimes. The “classical” regime, dominated by nearest-neighbor hopping between localized conducting puddles, manifests an activatedlike resistivity formula, and the quantized Hall insulator behavior. At very low temperatures \(T\), or farther from the critical point, a crossover occurs to a “quantum” transport regime dominated by variable range hopping. The latter is characterized by a different \(T\) dependence, yet the dependence on filling fraction is coincidentally hard to distinguish. [S0163-1829(99)02635-1]

However, counter to the expectation, \(\nu_0(T)\) does not scale as \(T^*\), but rather exhibits a linear dependence on \(T\)

\[
\nu_0(T) = aT + \beta.
\]

The resistivity law Eqs. (2) and (3) holds in various different samples, as well as in different transitions, including plateau-to-plateau transitions\(^5\) (with an appropriate definition of the analog of \(\rho_{xx}\)). The parameters \(\alpha\) and \(\beta\) are sample-dependent, and define a “saturation temperature” \(T_s = \beta/\alpha\), which ranges between 0.05 and ~0.5 K. Moreover, even if one ignores this saturation, attributing it to incomplete cooling of the carriers, a linear scaling of \(\nu_0(T)\) with \(T\) is inconsistent with any sensible theory for the quantum-critical behavior. It should be noted, that a partial set of curves obeying Eqs. (2) and (3) (corresponding to a restricted range of parameters) can be collapsed on a “traditional” scaling curve,\(^1\) with an exponent \(0 < \kappa < 1\). This observation raises a serious doubt concerning the interpretation of the data in earlier experimental works: the distinction between a pure power law \(T^\kappa\) with \(\kappa = 0.4\) ~0.5, and an alternative function that interpolated between \(\sim T\) and a constant, may turn out to be rather difficult. Nevertheless, a recent experimental result\(^8\) indicates a crossover from one behavior to another, which will be discussed later in this paper in more detail.\(^9\)

Another set of experimental observations that appear to be inconclusive involve the Hall resistance in the high-magnetic field insulator. While part of the data\(^2\) support the theoretical prediction of a “Hall insulator phase,”\(^18\) in which \(\rho_{xx}\) diverges in the limit \(T \rightarrow 0\) yet \(\rho_{xy}\) behaves as in a classical conductor (linearly dependent on the magnetic field \(B\)), other data\(^2\) exhibit a tendency to divergence of \(\rho_{xy}\) as well. Yet a third class of experimental data\(^4,7,10\) have established the existence of a “quantized Hall insulator” (QHI) behavior in the insulator close to a fundamental QH state (1/\(k\) with \(k\) an odd integer): in this regime, \(\rho_{xy}\) is not only finite as asserted in Ref. 18, but moreover maintains the quantized plateau value \(kh/\varepsilon^2\). This phenomenon is a specific manifestation of the “semicircle law,”\(^19\) which however extends beyond the range of validity expected from that theory. In particular, the observation of a QHI behavior in the nonlinear response regime\(^4\) indicates a surprising robustness of the phenomena, and has provided support to the idea that it is intimately related to the validity of duality symmetry.\(^16\)

Magnetotransport measurements in the vicinity of quantum Hall (QH) transitions provided over the recent years an extensive variety of data,\(^1\)–\(^10\) which stimulated a considerable confusion. The present paper suggests a way to settle the apparent disagreement between different experimental results.

The traditional point of view asserts that transport properties near the transitions between QH plateaux, and from a QH liquid to the insulator, should reflect the proximity to a second-order quantum phase transition. Correspondingly, the d.c. resistivity tensor \(\rho_{ij}\) at a given filling fraction \(\nu\) and temperature \(T\) should be described by a universal function \(f(X)\) of a single parameter

\[
\rho_{ij} = \rho_{ijc} f\left(\frac{\Delta \nu}{T}\right),
\]

where \(\Delta \nu = \nu - \nu_c\) (the deviation from the critical filling \(\nu_c\)), and \(\rho_{ijc}, \kappa\) are universal. Here, \(\kappa\) is a combination of critical exponents, \(\kappa = 1/\zeta x\), where \(\zeta\) is the dynamical exponent, and \(x\) characterizes the divergence of the correlation length near criticality: \(\xi \sim |\Delta \nu|^{-x}\). Theoretical studies have predicted \(x = 7/3\);\(^11\) experimentally, a number of groups have obtained data in remarkable consistency with the scaling ansatz, at different (integer as well as fractional) QH transitions.\(^12\) These results are further supported by data showing scaling at finite frequency\(^13\) and current,\(^14\) which moreover confirm the theoretical prediction \(x = 7/3\) and yield \(z = 1\) (indicating the relevance of Coulomb interactions\(^1\) near the quantum critical point).

The validity of the single parameter scaling form Eq. (1) has been, however, recently challenged.\(^6\) Motivated by an earlier work,\(^4\) which identified a surprisingly robust (duality) symmetry relating the current-voltage curves \(I(V)\) in the QH liquid phase \(|\Delta \nu| > 0\) (Ref. 15) to \(V(I)\) in the insulator at \(-\Delta \nu\), the parameter \(\Delta \nu\) was marked as a relevant scaling variable.\(^16\) The Ohmic resistivity plotted as a function of \(\Delta \nu\) is indeed fitted (in a wide range of parameters) by the formula

\[
\rho_{xx} = \frac{h}{e^2} \exp\left(-\frac{\Delta \nu}{\nu_0(T)}\right).
\]

The “classical” regime, dominated by nearest-neighbor hopping between localized conducting puddles, manifests an activatedlike resistivity formula, and the quantized Hall insulator behavior. At very low temperatures \(T\), or farther from the critical point, a crossover occurs to a “quantum” transport regime dominated by variable range hopping. The latter is characterized by a different \(T\) dependence, yet the dependence on filling fraction is coincidentally hard to distinguish. [S0163-1829(99)02635-1]
In a previous work, Shimshoni and Auerbach proposed a transport mechanism consistent with the above-described QHI phenomenon. The mechanism involves hopping across the junction between edge states surrounding nearest neighbors in a random network of 1/k-QH liquid puddles, carried out by quantum tunneling assisted by the temperature and current bias. Then, neglecting the quantum interference between different junctions in the network, it is proven that the Hall resistance is quantized at the value dictated by the QH liquid, irrespective of the details of the longitudinal resistance associated with the hopping processes in the junctions. Note that this network model can be extended to the case where the liquid puddles do not consist of a single type (a situation that is likely to be applicable in an insulating regime close to more than one fundamental QH state), in which case \( \rho_{xx} \sim B \) can be established (similarly to data in Ref. 2). It is later shown, that within this model one also obtains an activatedlike behavior similar to Eq. (2) [though with \( v_0(T) \) interpolating between \( \sim T \) and a constant in a way different from the linear expression Eq. (3)].

The linear dependence on \( \Delta \nu \) in this model is attributed to the relation between area fraction of the liquid and the barrier heights.

Underlying the above described model for the transport, there is an essential assumption of a finite \( L_\phi \), beyond which quantum interference terms are suppressed. As long as the size of a QH liquid puddle, around which electronic edge states are extended, is larger than \( L_\phi \)—the classical random resistor network model is justified. In this sense, the transport regime dominated by nearest-neighbor hopping processes is “classical.” This is even though the resistance associated with a single junction in the network (and given by a Landauer formula, where the two neighboring puddles are regarded as macroscopic reservoirs), is possibly dictated by quantum tunneling through the barrier.

The classical model of Ref. 20 is not obviously a unique scenario that yields a quantized Hall resistance away from the strict QH phase. To test this, in a recent work, Pryadko and Auerbach have examined the effect of quantum interference in the network on \( \rho_{xy} \), and found a deviation from the quantized value. In the case where \( L_\phi \) is much larger than a puddle size, \( \rho_{xy} \) vs \( B \) indicates an exponential divergence towards the insulating regime. A similar trend is indicated in other recent numerical data as well. This suggests that \( \rho_{xy} \) vs \( B \) is not necessarily the optimal destination of a hopping electron. Typically, randomly distributed electronic states that are close in energy are not close in real space. As a consequence, the dominant transport mechanism is variable range hopping (VRH). In this regime, a typical hop occurs between localized states separated by a distance \( R_h(T) \), which minimizes the exponential suppression of the hopping probability due to the difference in both energy and real space. Assuming a Coulomb gap in the density of states, this hopping length is given by

\[
R_h(T) \sim \left( \frac{\xi e^2}{ek_BT} \right)^{1/2};
\]

here \( \xi \) is the localization length, and \( \epsilon \) the dielectric constant. The resulting expression for the longitudinal resistance in the insulator is

\[
\rho_{xx} \sim \rho_0 \exp \left[ \frac{T_0}{T} \right]^{1/2}, \quad T_0 = \frac{e^2}{k_B \xi \epsilon}. \tag{5}
\]

Similarly, in the QH phase Eq. (5) holds for \( 1/\rho_{xx} \); to avoid confusion, in the rest of the paper the expressions for \( \rho_{xx} \) correspond to the insulator.

To convert Eq. (5) into a dependence on the filling fraction, note that the localization length \( \xi \) (which by definition describes a typical cluster over which an electronic state is extended), coincides with the correlation length, which tends to diverge near the transition:

\[
\xi = \xi_0 \left( \frac{\Delta \nu}{v_c} \right)^{-x} \tag{6}
\]

(where \( \xi_0 \) is the value of \( \xi \) deep in the localized phase). The critical behavior Eq. (6) is valid for \( \Delta \nu < v_c \), so that \( \xi \ll \xi_0 \). On the other hand, the mechanism of VRH dominates the transport as long as \( \xi \) is finite and smaller than \( L_\phi \). Provided the latter is a few orders of magnitude larger than \( \xi_0 \), there is a range of parameters where Eq. (5) holds in coincidence with Eq. (6). As a result, one obtains

\[
\rho_{xx} \sim \rho_0 \exp \left[ \frac{C|\Delta \nu|^{1/2}}{T} \right], \quad C = \frac{e^2}{k_B \xi_0 \epsilon v_c}. \tag{7}
\]

In the regime where Eq. (7) is applicable, the experimental data exhibit three prominent features: (a) the scaling form Eq. (1) is recovered with \( \kappa = 1/\alpha = 0.43 \) (given that indeed \( x = 7/3 \)), and \( f(X) \sim |\Delta \nu|^{1/2} \); (b) at a given \( \Delta \nu \),

\[
\ln \rho_{xx}(T) \sim T^{-1/2}, \tag{8}
\]

and (c) isotherms plotted as a function of \( \nu \) are of the form

\[
\ln \rho_{xx}(\nu) \sim |\Delta \nu|^{1.15}. \tag{9}
\]

Comparing Eq. (9) with the empirical resistivity law (2), we observe that by mere coincidence (which stems from the specific value of the exponent \( x \)), the two functional forms are practically indistinguishable. Similarly, it is hard to distinguish the temperature dependence Eq. (8) from the fit to \( T^{-\kappa} \) employed in Ref. 8; it is suggested that the VRH scenario is, in fact, a more appropriate basis for interpretation of the data in the low-\( T \) regime.

As mentioned above, the VRH scenario is consistent in the regime where the transport is quantum coherent, namely for \( \xi \ll L_\phi \). Hence, the quantum regime terminate once \( \xi \) approaches \( L_\phi \) due to either increase of temperature, or the divergence of \( \xi \) sufficiently close to \( v_c \). To estimate the boundary of the corresponding region in parameter space, an explicit expression for \( L_\phi \) is needed. It turns out that in the VRH regime, the length scale that plays the role of a dephasing length is the hopping length \( R_h(T) \) [Eq. (4)]. This implies that a crossover to a “classical” transport regime occurs at \( T \sim T_0 \) [where \( T_0 \) is defined in Eq. (5)]. Note that this
criterion is consistent with the observation, that for \( T > T_0 \) the longitudinal resistivity no longer indicates the exponential divergence characteristic of strong localization. Employing Eq. (6), we conclude that for a fixed \( \nu \), a crossover to the classical regime occurs at a temperature \( T_x \), where

\[
T_x \sim T_0 - \frac{e^2}{k_B\epsilon\xi_0} \left( \frac{|\Delta \nu|}{\nu_c} \right)^x.
\]

Alternatively, for a fixed \( T \), the crossover occurs at \( |\Delta \nu|_x \), where

\[
|\Delta \nu|_x \sim \left( \frac{\epsilon\xi_0 k_BT}{e^2} \right)^{1/x}.
\]

As argued in Ref. 26, the latter expression defines the width of the peaks in \( \sigma_{xx} \) near QH transitions. However, it should be emphasized that (at a fixed \( T \)) critical scaling of the data is expected to hold outside this width, while \( |\Delta \nu| \leq |\Delta \nu|_x \) corresponds to a classical transport regime.

I next show that the data that clearly manifest the activated-like behavior resistivity law (2) and (3) (Ref. 6) and a QHI behavior (Refs. 7 and 10), mostly correspond to the classical regime by the criterion suggested above. A quantitative estimate of \( |\Delta \nu|_x \) from Eq. (11) is possible provided the “bare” localization length \( \xi_0 \) is known. Unfortunately, this parameter cannot be extracted independently from the available information about the samples. However, the fact that the integer QH effect is observed indicates that the single-electron states are localized over a length scale at least as large as the magnetic length \( l = (\hbar c / eB)^{1/2} \). Hence, the insertion \( \xi_0 \sim l \) provides a minimal estimate of \( |\Delta \nu|_x \) for a given \( T \). In Ref. 6, close to the critical field in the InGa1−xAs/InP sample \( (B_c = 2.14 \text{ T}, \text{ corresponding to } \nu_c = 0.562 \text{ and carrier density } n = 3 \times 10^{10} \text{ cm}^{-2}) \), one gets \( l \sim 170 \text{ Å} \). The implied lower bound on the width of the classical regime is \( (\Delta \nu)_x / \nu_c \approx \pm 0.2 \) for the highest temperature isotherm (\( T = 2.21 \text{ K} \)), and \( (\Delta \nu)_x / \nu_c \approx \pm 0.1 \) for \( T = 0.3 \text{ K} \). Comparing with the data, it turns out that the range of \( \nu 's \) where \( \ln \rho_{xx} \) vs \( \nu \) is strictly linear is not much larger than this lower bound. A more conclusive statement can be made regarding the quantized Hall resistance data of Hilke et al. in Ref. 7: there, \( l \approx 130 \text{ Å} \), which implies that at the lowest displayed temperature (\( T \approx 0.3 \text{ K} \)), the classical regime extends at least within \( \Delta B / B_c \approx 0.09 \). Indeed, this estimate implies an upper field \( B_{\nu} = B_c \times 1.09 \), which roughly coincides with the field at which the \( \rho_{xy} \) data terminate (due to insufficient accuracy of the measurement). Note that the range of observed QHI increases with \( T \) or with an increased current bias, as long as a plateau in the QH phase is preserved. Beyond a certain \( T \), the quantization in the insulator is destroyed at the same time with the entire plateau, due to excitations to higher Landau levels.

To further test the central arguments of this paper, one should examine experimental data that extend over a wide enough range of temperatures below and above the crossover point \( T_x \). The classical and quantum regimes are then clearly distinct in terms of the \( T \) dependence of \( \rho_{xx} \) for a given \( \nu \). The functional dependence on \( \nu \) is, however, nearly identical: \( \ln \rho_{xx} \) is expected to be approximately linear in \( \nu \) in a wide range of parameters extending over both regimes. This is possibly a major source of confusion in the literature. In particular, in Ref. 8 a crossover in the \( T \) dependence is clearly observed, however the single crossover point identified there (\( T_x \approx 0.1 \text{ K} \)) is an average over a range of filling fractions. Similarly, it is possible that in Ref. 6 as well, the outskirts of the range of \( \Delta \nu \) indicating \( \ln \rho_{xx} \sim \nu \) extend into the quantum regime. The formula Eq. (3) is then not necessarily the only possible fit of the slope.

To summarize, I propose an interpretation of the extensive set of data close to QH transitions, which distinguishes two conduction regimes. The classical regime is established closer to the critical point and at relatively high \( T \). It is dominated by hopping between nearest-neighbor hopping between conducting QH puddles, whose typical size is larger than the dephasing length \( L_0 \). Hence, the transport coefficients do not depend on \( L_0 \), but rather on the details of the narrow junctions separating the puddles. When mapped to a QH liquid-to-insulator transition, the characteristic behavior of the resistivity tensor in this regime is an activatedlike \( \rho_{xx} \), and quantization of \( \rho_{xx} \) in the insulator. The quantum regime is established at lower \( T \) and farther from the critical point, where the transport is dominated by VRH. The limit of validity of VRH [corresponding to \( \xi \sim R_h(T) \), where \( R_h(T) \) is identified with \( L_0 \)], provides an estimate of the boundary between the regimes [Eqs. (10) and (11)]. The classical and quantum regimes are very hard to distinguish by \( \nu \) dependence of \( \ln \rho_{xx} \sim (\Delta \nu - \nu)^{1/2} \), respectively. It is predicted that the crossover between them should be more clearly indicated by a change in the \( T \) dependence, accompanied by a deviation of \( \rho_{xx} \) in the insulator from a quantized plateau.

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9. The two distinct transport regimes were also identified by M. Furlan, Phys. Rev. B 57, 14818 (1998).
15. In the case where the QH liquid state is fractional, $\nu$ is replaced by the filling fraction of composite fermions; see Ref. 16.
27. Recent experimental data support the critical scaling of the VRH parameter $T_0$ [S. Murphy (unpublished)]. Similar critical behavior was observed also near the Anderson transition in three-dimensional insulators, see T. G. Castner in Ref. 24(b).