New geometries for spin and charge pumping

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Workshop on quantum pumping, 7 January 2007

Outline

- Spin pump turnstile

  [with Catherine Fricot, Phys. Rev. B **71**, 041303 (R) (2005)]

- Adiabatic quantum pumping with quantized radiation fields

  [with Björn Trauzettel, unpublished]

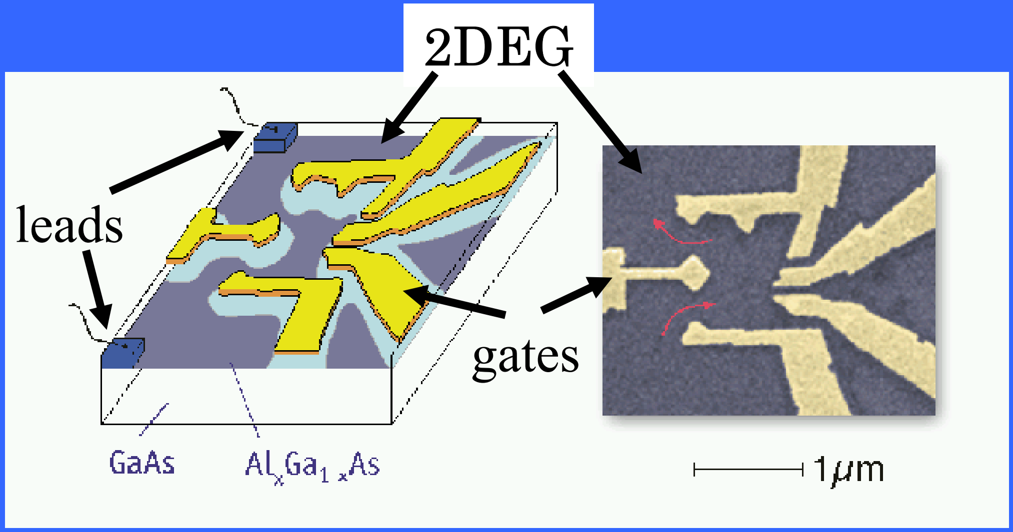
- Conclusion and Outlook

1. Spin pump turnstile

[M. Blaauboer and C.M.L. Fricot, Phys. Rev. B **71**, 041303 (R) (2005)]

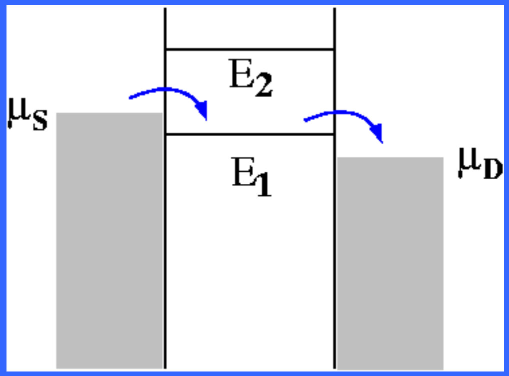
Idea: propose a device which can create a pumped spin-polarized current in a controlled way

Geometry: quantum dot



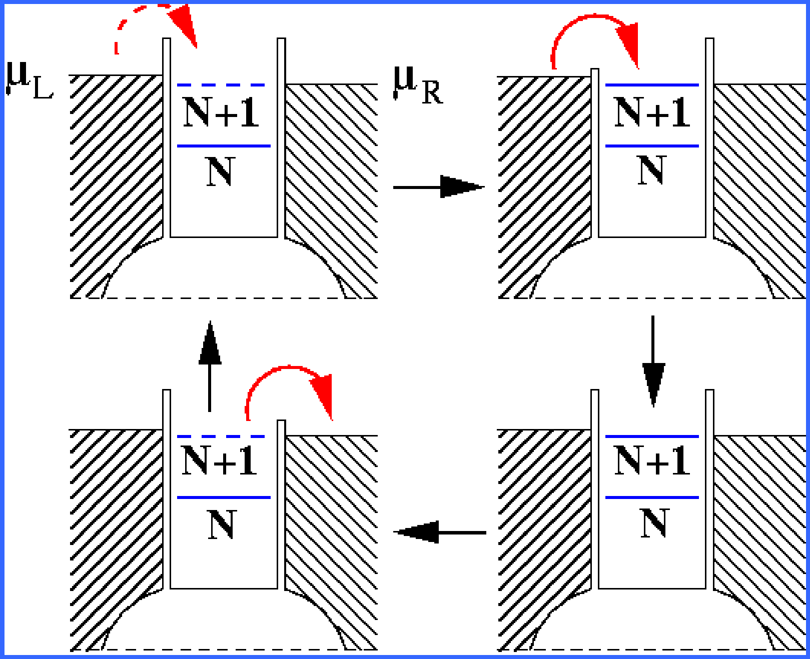
externally controllable "artificial atom"

Energy level spectrum



1991/1992: Charge turnstile

[L.P. Kouwenhoven *et al*, Phys. Rev. Lett. **67**, 1626 (1991);
H. Pothier *et al*, Europhys. Lett. **17**, 249 (1992)]



- Coulomb blockade regime
- Current quantized in units of eω
- 'Classical pump': tunneling, but no quantum interference

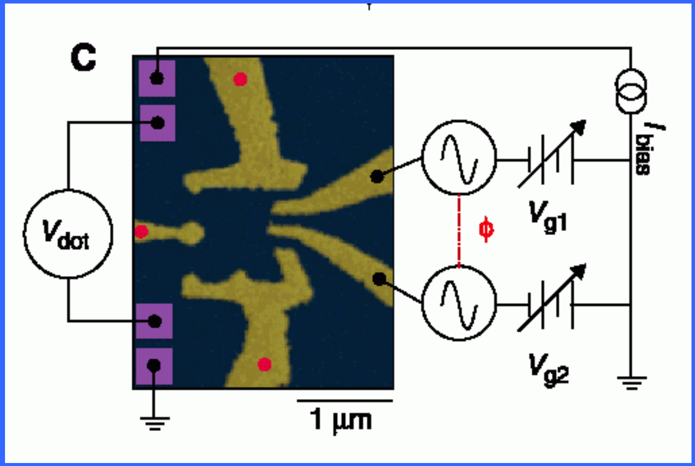
Quantum charge "pump"

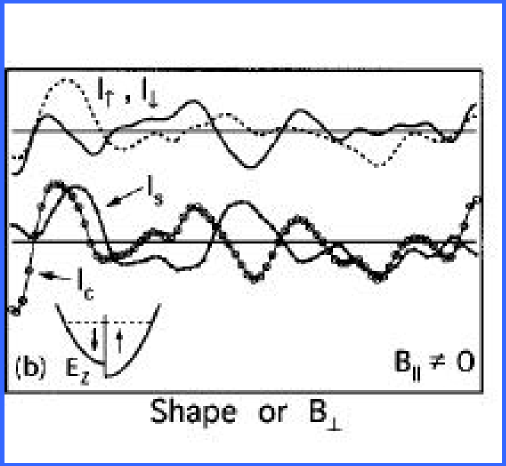
[Switkes *et al.*, Science **283**, 1905 (1999)]

Quantum spin "pump"

[Watson *et al.*, PRL **91**, 258301 (2003)]

Experimental set-up, open quantum dot





Red gates control the conductance of the point contacts

Black gates are used for pumping

adiabatic transfer of spin-without-charge

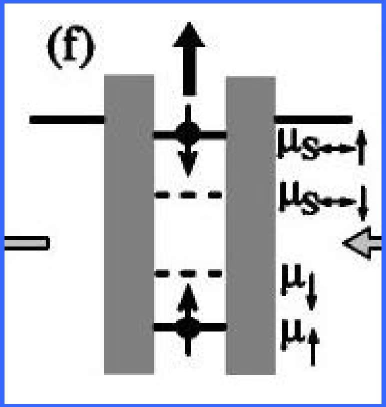
Spin turnstile

- Controllable **double-sided** and **bipolar** spin filter
- Spin injection on demand
- Spin current quantized in units of 2 spins/τ

System: quantum dot in magnetic field
in Coulomb blockade regime

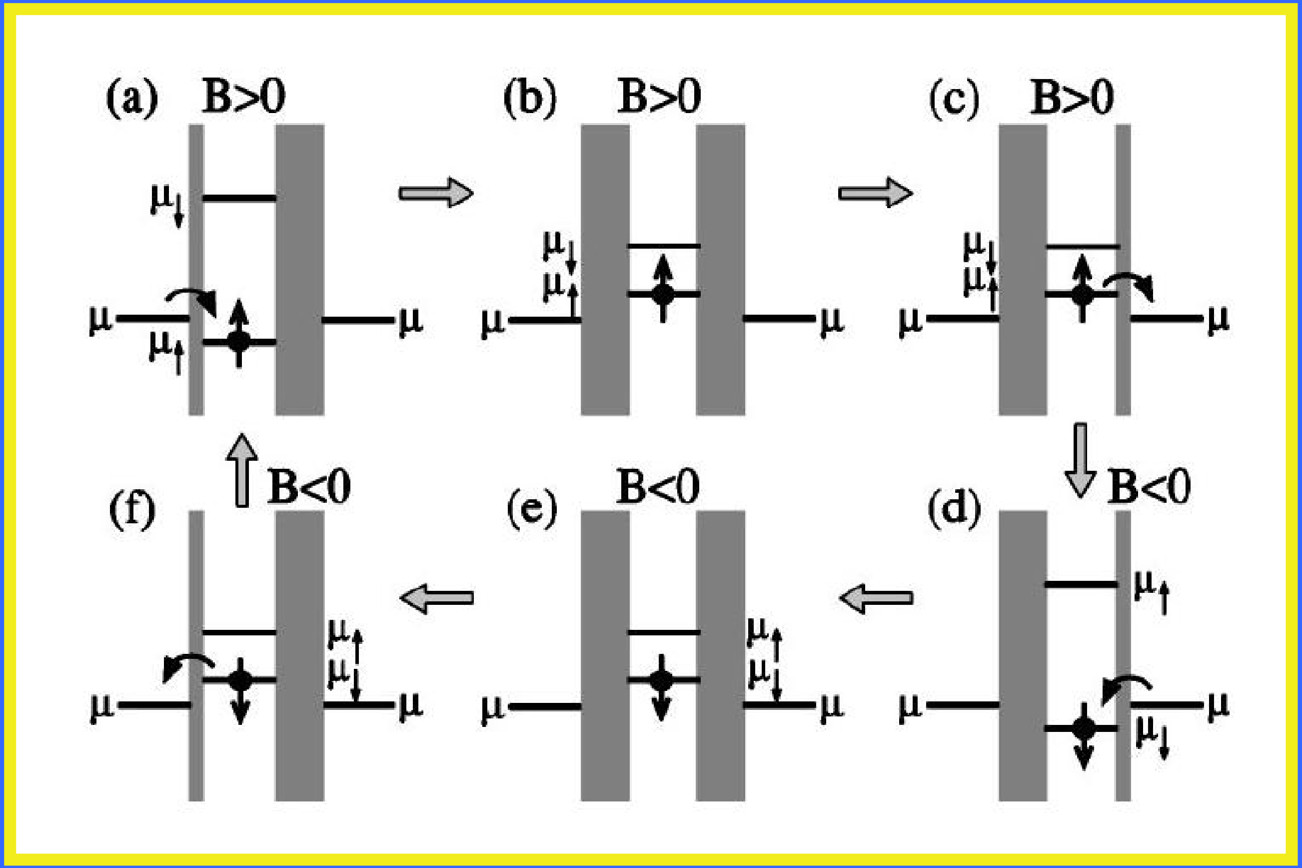
Pumping parameters:

1) **difference in tunnel coupling** to the left and right lead
2) either **magnetic field or additional gate voltage**



Level occupation:
empty, single, double (singlet)

$$\mu_{\uparrow(\downarrow)} = E_{\uparrow(\downarrow)} - E_0 = E_{\uparrow(\downarrow)}$$
$$\mu_{S\leftrightarrow\uparrow(\downarrow)} = E_S - E_{\uparrow(\downarrow)} = E_{\downarrow(\uparrow)} + E_C$$



Analysis of relevant time scales

- Raising/lowering time of tunnel barriers τ_{raise} **fast**
- Tunneling time τ_{tunnel} for electron to enter/leave the dot
- Switching time of magnetic field τ_{switch}
- Spin-flip time on dot $\tau_{spinflip}$

Requirements: τ_{tunnel} and τ_{switch} **as fast as possible** (maximize current), but also **reliable** transfer of spin (maximize **spin** current)

Probability of adiabatic transfer

Hamiltonian

$$\mathcal{H} = \frac{(\vec{p} - e\vec{A})^2}{2m^*} - \frac{1}{2}E_Z\sigma + \frac{1}{2}m^*\omega_0^2(x^2 + y^2)$$

Fock-Darwin energy-level spectrum

$$E_{n,l,\sigma} = (2n + |l| + 1)\hbar\omega - \frac{1}{2}l\hbar\omega_c - \frac{1}{2}E_Z\sigma$$

$$\omega \equiv \sqrt{\omega_0^2 + \omega_c^2/4}$$

$$\omega_c = eB/m^*$$

Lowest orbital levels: n=0, l=0
n=0, l=1

No spin interaction:

$$[\mathcal{H}, L_z] = 0$$

$$[\mathcal{H}, S_z] = 0$$

→ no level transitions with spin flip

Spin-orbit interaction

$$\mathcal{H}_R = \frac{\alpha_R}{\hbar}[\vec{\sigma} \times (\vec{p} - e\vec{A})]_z$$

$$[\mathcal{H}_{\mathcal{R}}, J_z] = 0$$

$$[\mathcal{H}_{\mathcal{R}}, S_z] \neq 0$$

adiabatic transfer with spin-flip in principle possible

Modified energy level spectrum (perturbation theory)

$$\tilde{E}_{0,0,1\,(0,1,-1)} = \frac{3}{2}\hbar\omega - \frac{1}{4}\hbar\omega_c - (+)\frac{1}{2}\left[(\hbar\omega - \frac{1}{2}\hbar\omega_c + E_Z)^2 + \tilde{\alpha}_R(\hbar\omega - \frac{1}{2}\hbar\omega_c)\right]^{\frac{1}{2}}$$

No level crossing during switching of magnetic field

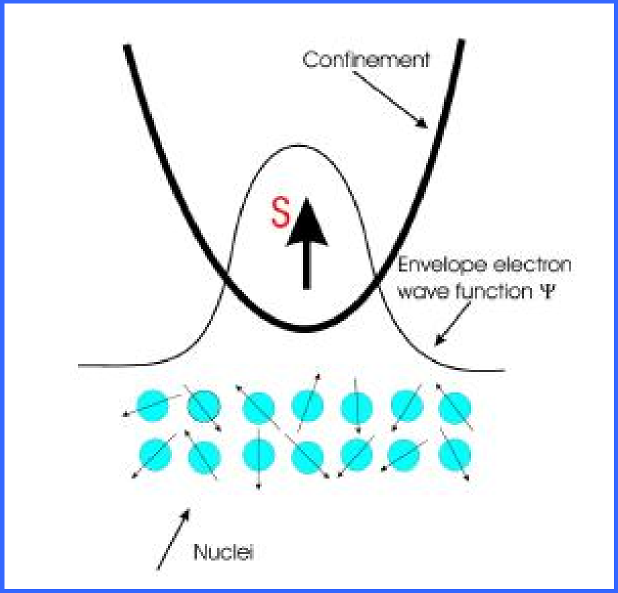
Hyperfine interaction

$$\mathcal{H}_{\rm hyp} = AS_zI_z + \frac{1}{2}A(S_+I_- + S_+I_-)$$

$$[\mathcal{H}_{\rm hyp}, L_z] = 0$$

$$[\mathcal{H}_{\rm hyp}, S_z] \neq 0$$

adiabatic transfer with spin-flip in principle possible



Overhauser field $B \sim A/\sqrt{N}$

Adiabatic transfer time:

$$T_{\rm ad} \gg \frac{\hbar N^2 |g^*| \mu_B B}{N_{\rm eff}^2 A^2}$$

For $$\begin{aligned} \tau_{\text{switch}} &\ll T_{\text{ad}} \\ \tau_{\text{switch}} &\leq \tau_{\text{tunnel}} \end{aligned}$$ tunneling time dominates

Pumped charge and spin currents:

$$\begin{aligned} I_c &= \frac{e}{\tau}(n_{\uparrow,R} + n_{\downarrow,R} - n_{\uparrow,L} - n_{\downarrow,L}) \\ I_s &= \frac{1}{\tau}(n_{\uparrow,R} - n_{\downarrow,R} - n_{\uparrow,L} + n_{\downarrow,L}) \end{aligned}$$

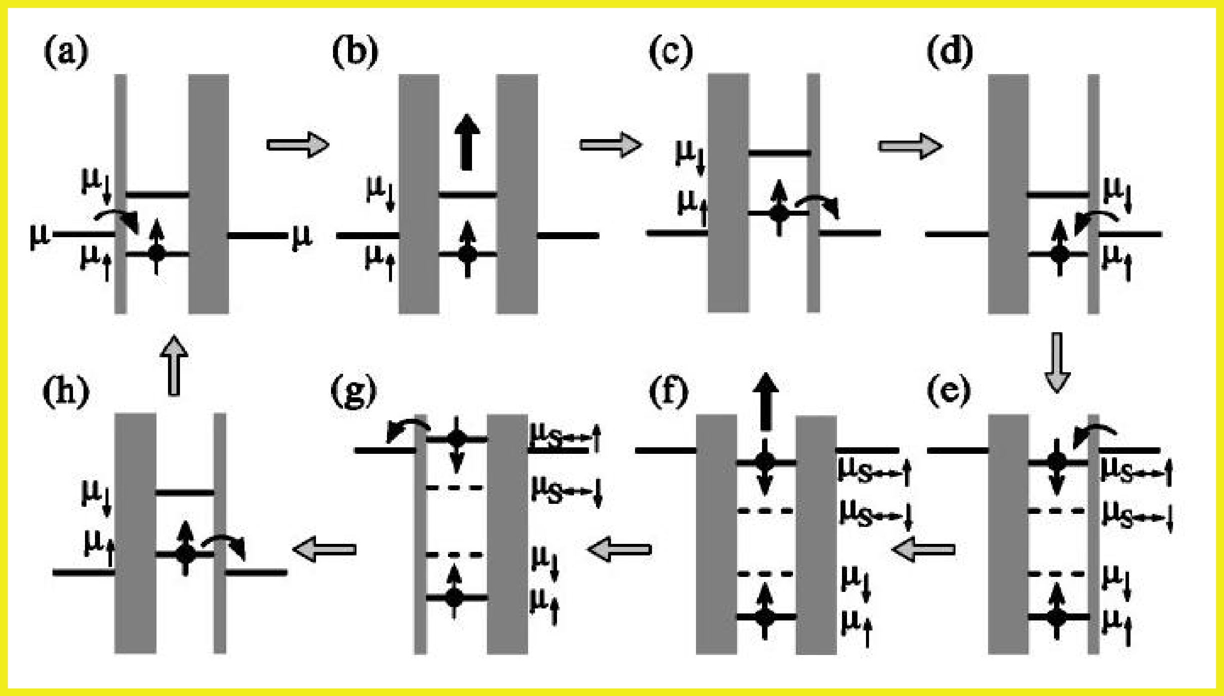
Including spin-flip scattering

$$\dot{\rho}_i = W_{ij}\rho_j + W_{ik}\rho_k - (W_{ji} + W_{ki})\rho_i$$

$\Rightarrow$

$$n_{\uparrow,R} = \frac{W_{\uparrow 0,L} W_{0\uparrow,R} W_{\uparrow\downarrow}^2}{\prod_{M=L,R}(W_{0\uparrow,M} W_{\uparrow\downarrow} + W_{\uparrow 0,M}(W_{\uparrow\downarrow} + W_{\downarrow\uparrow}))}$$

Alternative: pumping with an additional gate voltage



- Singlet ground-state $\rightarrow$ parallel magnetic field
- Singlet-triplet spin-flip time $\gg \tau_{tunnel}$
- **Disadvantage**: less efficient cycle
- **Advantage**: fast pulsed gate voltages available ($\leq$1 ns)
  [Phase information can be preserved]

2. Adiabatic quantum pumping of charge with quantum-mechanical EM fields

[With Björn Trauzettel (Basel)]

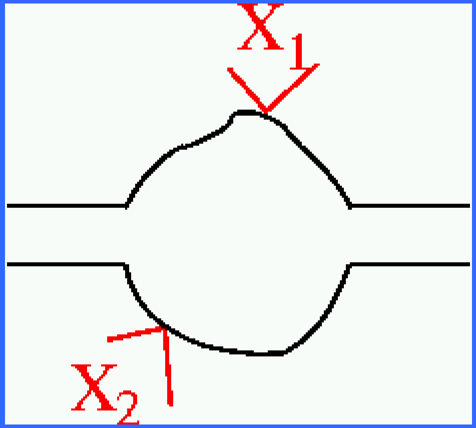
Main question we are trying to answer :

if the pumping fields are entangled, can we detect a (unique) **signature** of this entanglement in the pumped current?

In particular: can the pumped current be used to **distinguish** between **entangled** and **non-entangled** radiation?

Answer: The pumped current can only be used to detect some specific entangled states, not arbitrary ones (for example, Bell states)

Quantum pumping with classical fields



Pumping parameters :

$$\mathbf{X_1}(t) = \mathbf{X_1} + \mathbf{2}\,\delta\mathbf{X_1}\cos(\omega t - \phi_1)$$
$$\mathbf{X_2}(t) = \mathbf{X_2} + \mathbf{2}\,\delta\mathbf{X_2}\cos(\omega t - \phi_2)$$

Pumped current into lead α :

$$\mathbf{I}_\alpha = \frac{\omega e}{2\pi}\,\delta X_1 \delta X_2 \sin\phi \,\mathrm{Im}\left[\sum_\beta \frac{\partial S^*_{\alpha\beta}}{\partial X_1}\frac{\partial S_{\alpha\beta}}{\partial X_2}\right] \qquad (1)$$

Brouwer, PRB **58**, R10135 (1998)

valid for :
- **weak pumping** (linear response in δX_1 and δX_2)
- **adiabatic limit** $(\omega \ll \tau^{-1}_{dwell})$

Scattering matrix for adiabatic weak pumping :

$$\begin{aligned} S(t) &= S(X_1(t), X_2(t)) \\ &\approx S(X_1, X_2) + \sum_{j=1}^{2} \frac{\partial S}{\partial X_j}\delta X_j(t) \\ &= S(X_1, X_2) + S_+ e^{-i\omega t} + S_- e^{i\omega t} \end{aligned}$$

$$S_\pm \equiv \sum_{j=1}^{2} \delta X_j \, e^{\pm i\phi_j} \frac{\partial S}{\partial X_j}$$

Pumped current into lead α :

$$\mathbf{I}_\alpha = \frac{\omega e}{2\pi}\left[T_{+,\alpha} - T_{-,\alpha}\right] \qquad (2)$$

Moskalets and Büttiker, PRB **66**, 035306 (2002)

with $T_{\pm\alpha} = \sum_\beta |S_{\pm,\alpha\beta}|^2$ the total probability for an electron which is scattered into lead α to absorb (+) or emit (-) an energy quantum ħω

Eqns. (1) and (2) are equivalent

Adiabatic quantum pumping with quantum-mechanical radiation fields

Time-dependent scattering matrix for two-photon pumping fields :

$$\mathbf{S}(t,t') = \mathbf{S}_0(\Delta t)\,\mathcal{T}\,e^{\,i\sum_{j=1}^{2}\Lambda_j \int_{t'}^{t} d\tau\,\left(\mathbf{a}_j(\tau)+\mathbf{a}_j^{\dagger}(\tau)\right)}$$

with $\begin{array}{l} a_j(\tau) \equiv a_j\,e^{-i\omega t} \\ a_j^{\dagger}(\tau) \equiv a_j^{\dagger}\,e^{i\omega t} \end{array}$ quantum-mechanical photon annihilation (creation) operator

Λ_j : real symmetric matrix that contains information on how the shape of the mesoscopic system is varied by photon radiation

$\mathrm{E}t \equiv t - t'$

Weak pumping : only linear terms in $a_j^{(\dagger)}$ are kept
Adiabatic regime : $\omega\Delta t$ ¨ 1

$$\mathbf{S}(t,t') \approx \mathbf{S}_0(\Delta t)\left[1 + i\Delta t \sum_{j=1}^{2}\Lambda_j\left[a_j^{\dagger}\,e^{i\omega T} + a_j\,e^{-i\omega T}\right]\right]$$

$T \equiv (t+t')/2$

Wigner transform

$$S(\epsilon, T) = \int_{-\infty}^{\infty} d(\Delta t)\, e^{-i\epsilon\Delta t} S(T + \frac{\Delta t}{2}, T - \frac{\Delta t}{2})$$

Quantum-mechanical radiation fields :

$$S(\epsilon, T) = S_0 - \sum_{j=1}^{2} a_j \Lambda_j \frac{\partial S_0}{\partial \epsilon} e^{-i\omega T} - \sum_{j=1}^{2} a_j^\dagger \Lambda_j \frac{\partial S_0}{\partial \epsilon} e^{i\omega T}$$

Classical fields :

$$S(t) = S(X_1, X_2) + \sum_{j=1}^{2} \delta X_j e^{i\phi_j} \frac{\partial S}{\partial X_j} e^{-i\omega t} + \sum_{j=1}^{2} \delta X_j e^{-i\phi_j} \frac{\partial S}{\partial X_j} e^{i\omega t}$$

Equivalent expressions under identification :

$$S(X_1, X_2) \leftrightarrow S_0$$
$$\delta X_j e^{i\phi_j} \frac{\partial S}{\partial X_j} \leftrightarrow -a_j \Lambda_j \frac{\partial S_0}{\partial \epsilon}$$
$$\delta X_j e^{-i\phi_j} \frac{\partial S}{\partial X_j} \leftrightarrow -a_j^\dagger \Lambda_j \frac{\partial S_0}{\partial \epsilon}$$

Calculate current (2) averaged over photon fields :

$$\langle I_\alpha \rangle = \frac{e\omega}{2\pi} \left\{ \left[\langle a_1 a_2^\dagger \rangle - \langle a_1^\dagger a_2 \rangle \right] \sum_\beta \left[\frac{\partial S_{0,\alpha\beta}}{\partial X_1} \frac{\partial S_{0,\alpha\beta}^*}{\partial X_2} \right] + \left[\langle a_2 a_1^\dagger \rangle - \langle a_2^\dagger a_1 \rangle \right] \sum_\beta \left[\frac{\partial S_{0,\alpha\beta}}{\partial X_2} \frac{\partial S_{0,\alpha\beta}^*}{\partial X_1} \right] \right\}$$

Application to few-photon states

$$\langle \mathbf{I}_\alpha \rangle = \frac{e\omega}{2\pi} \left\{ \left[\langle a_1 a_2^\dagger \rangle - \langle a_1^\dagger a_2 \rangle \right] \sum_\beta \left[\frac{\partial S_{0,\alpha\beta}}{\partial X_1} \frac{\partial S^*_{0,\alpha\beta}}{\partial X_2} \right] + \left[\langle a_2 a_1^\dagger \rangle - \langle a_2^\dagger a_1 \rangle \right] \sum_\beta \left[\frac{\partial S_{0,\alpha\beta}}{\partial X_2} \frac{\partial S^*_{0,\alpha\beta}}{\partial X_1} \right] \right\}$$

1) **Vacuum state** $|0,0\rangle \rightarrow \langle I_\alpha \rangle = 0$ (trivial)

2) **Non-entangled state** of a single photon in mode 1 and no photon in mode 2 $|1,0\rangle \rightarrow \langle I_\alpha \rangle = 0$ (no pumping for a single parameter)

3) **Entangled state** $\gamma_1 |1,0\rangle + \gamma_2 |0,1\rangle \rightarrow$

$$\langle \mathbf{I}_\alpha \rangle = \frac{2e\omega}{\pi} \, \mathrm{Im}[\gamma_1^* \gamma_2] \, \mathrm{Im} \left[\sum_\beta \frac{\partial S_{0,\alpha\beta}}{\partial X_1} \frac{\partial S^*_{0,\alpha\beta}}{\partial X_2} \right]$$

- $\gamma_1 = 0$ or $\gamma_2 = 0 \rightarrow \langle I_\alpha \rangle = 0$
- $\langle I_\alpha \rangle \neq 0$ requires **$\mathrm{Im}[\gamma_1^* \gamma_2] \neq 0$**

Fails for Bell state: $(|1,0\rangle + |0,1\rangle)/\sqrt{2} \rightarrow \langle I_\alpha \rangle = 0$

cannot be distinguished from a non-entangled state

Conclusion and Outlook

Search for interesting quantum systems where:

- Novel aspects of quantum pumping can be tested (spin effects, entanglement effects)
- There is good hope for experimental verification

Possible candidate: phase pumping in N-S system