Semiclassical Theory of a quantum pump

Piet Brouwer
Laboratory of Atomic and Solid State Physics
Cornell University

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Packard Foundation

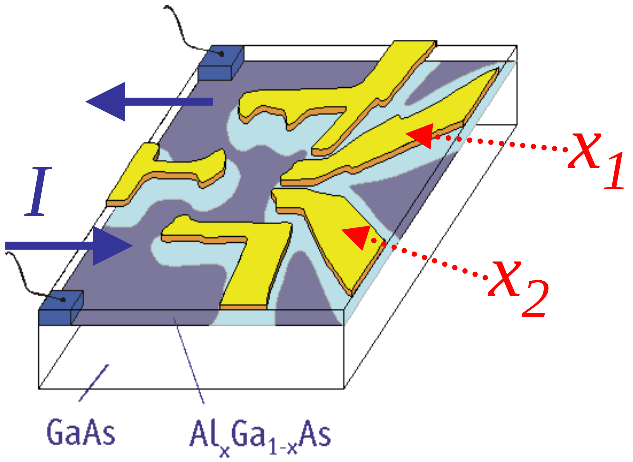
With: Saar Rahav

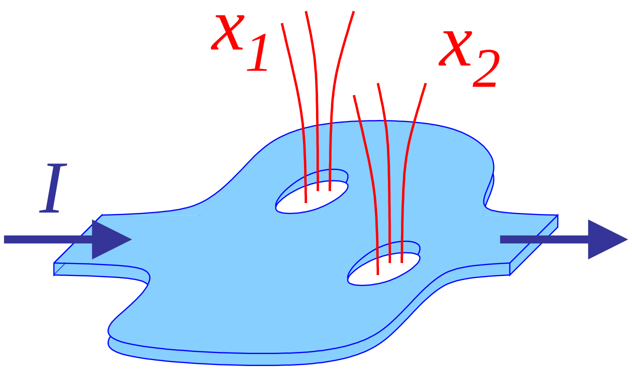
Quantum pump

A "quantum pump" is an electron pump based on quantum interference only

Examples:

- quantum dot with shape-distorting gate voltages x_1, x_2
- quantum dot penetrated by Aharonov-Bohm fluxes x_1, x_2



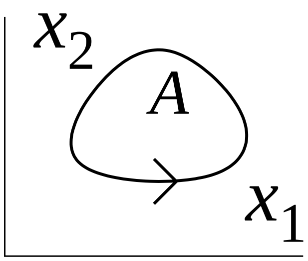


Quantum pump

Adiabatic regime: x_1, x_2 slow on scale of dwell time τ_D

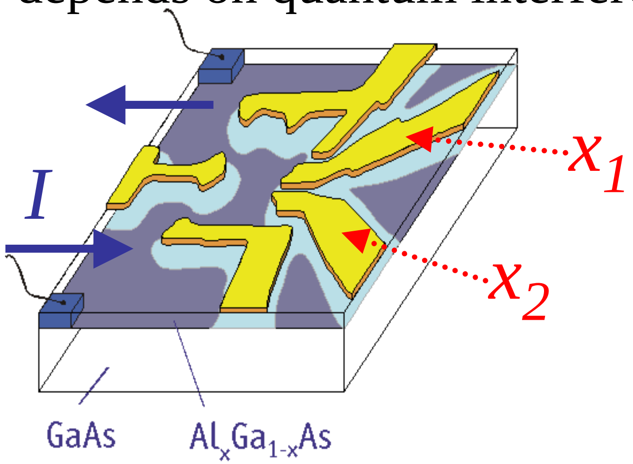
Charge pumped in one cycle:

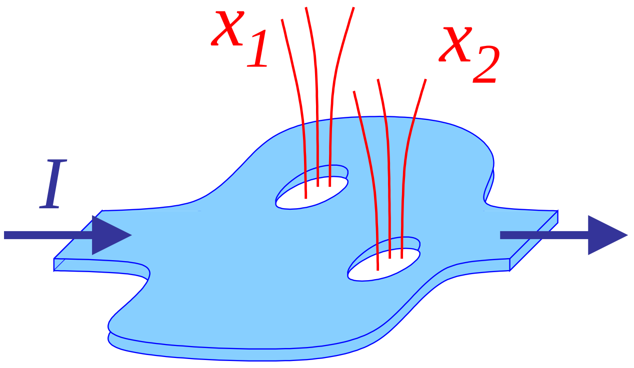
$$Q = \frac{e}{\pi} \int_A dx_1 dx_2 \sum_n \mathrm{Im} \frac{\partial S_{mn}}{\partial x_2} \frac{\partial S^*_{mn}}{\partial x_1}.$$



- *Q* is sample specific;
  depends on quantum interference.

Büttiker, Thomas, Prêtre (1994)
PWB (1999)





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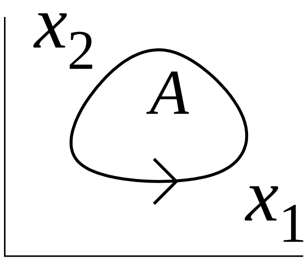
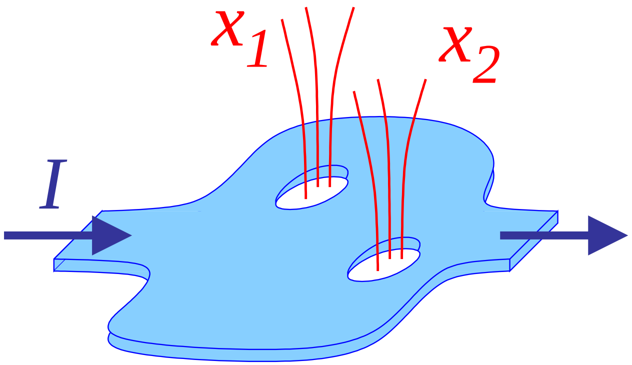


- *Q* is sample specific;
  depends on quantum interference.

- Probability distribution of *Q* can be calculated using Random Matrix Theory.



Quantum pump

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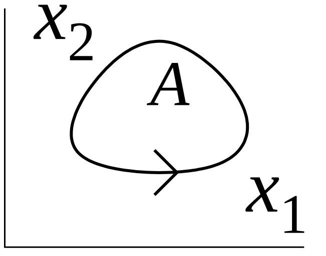

$$Q = \frac{e}{\pi}\int_A dx_1 dx_2 \sum_n \mathrm{Im}\,\frac{\partial S_{mn}}{\partial x_2}\frac{\partial S^*_{mn}}{\partial x_1}.$$

- Q is sample specific;
  depends on quantum interference.

- Probability distribution of Q can be calculated using Random Matrix Theory.

$$\langle Q^2 \rangle_{\mathrm{RMT}} = C_1 C_2 \left[\frac{e\tau_D \sin(\theta_1 - \theta_2)}{2}\right]^2$$

- $x_1 = x_{10}\cos(\omega t + \theta_1)$
- $x_2 = x_{20}\cos(\omega t + \theta_2)$
- Bilinear response

τ_D: mean dwell time

PWB (1999)

Quantum pump

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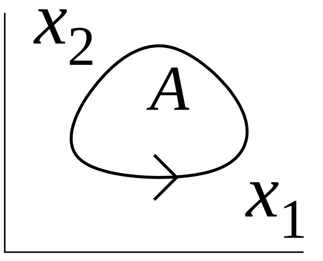

$$Q = \frac{e}{\pi} \int_A dx_1 dx_2 \sum_n \mathrm{Im} \frac{\partial S_{mn}}{\partial x_2} \frac{\partial S_{mn}^*}{\partial x_1}.$$

- Q is sample specific;

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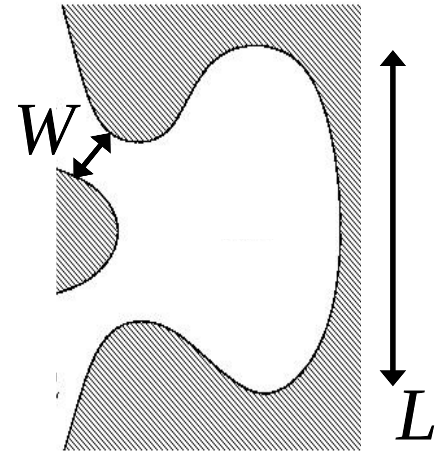
When can Random Matrix Theory be used?

Random Matrix Theory

Dwell time $\tau_D >>$ ergodic time τ_{erg}

quantum dot geometry

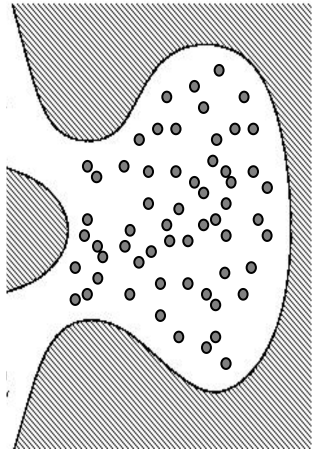


$\tau_D \sim \tau_{erg} L/W$

L: dot size

W: contact width

Dot with point scatterers: $\tau_{erg} = L^2/D$



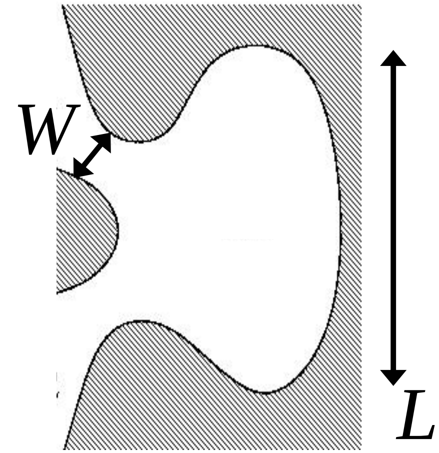
D: diffusion constant

Microscopic proof: Efetov (1983)

Random Matrix Theory

Dwell time τ_D >> ergodic time τ_{erg}

quantum dot geometry



$\tau_D \sim \tau_{erg} L/W$

L: dot size

W: contact width

Ballistic quantum dot: $\tau_{erg} = L/v_F$

v_F: Fermi velocity

Second condition:

Dwell time τ_D >> Ehrenfest time $\tau_E = \lambda^{-1}\ln(k_F L)$

separation α $e^{\lambda t}$

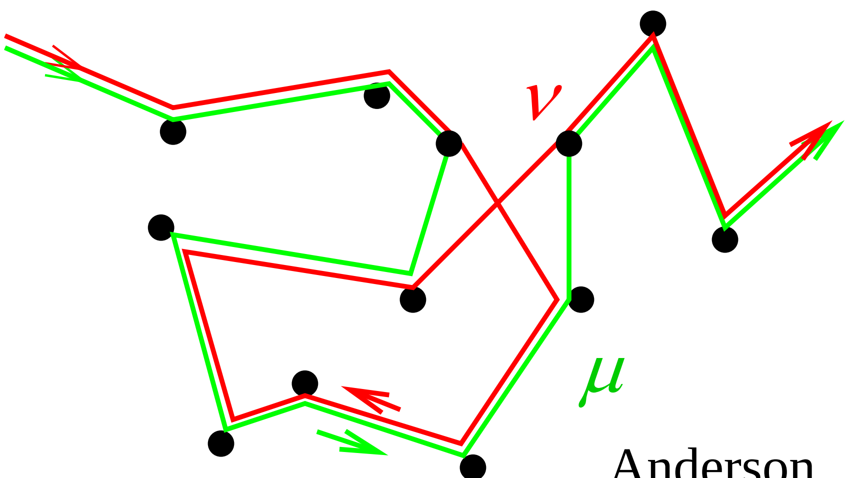
k_F: Fermi wavenumber

$\lambda \sim \tau_{erg}^{-1}$: Lyapunov exponent

Detour: Weak localization

Negative correction $\langle \delta G \rangle$ to ensemble averaged conductance
Origin: quantum interference

$$G = \sum_{\mu} |A_{\mu}|^2 + \sum_{\mu \neq \nu} A_{\mu} A_{\nu}^{*}$$

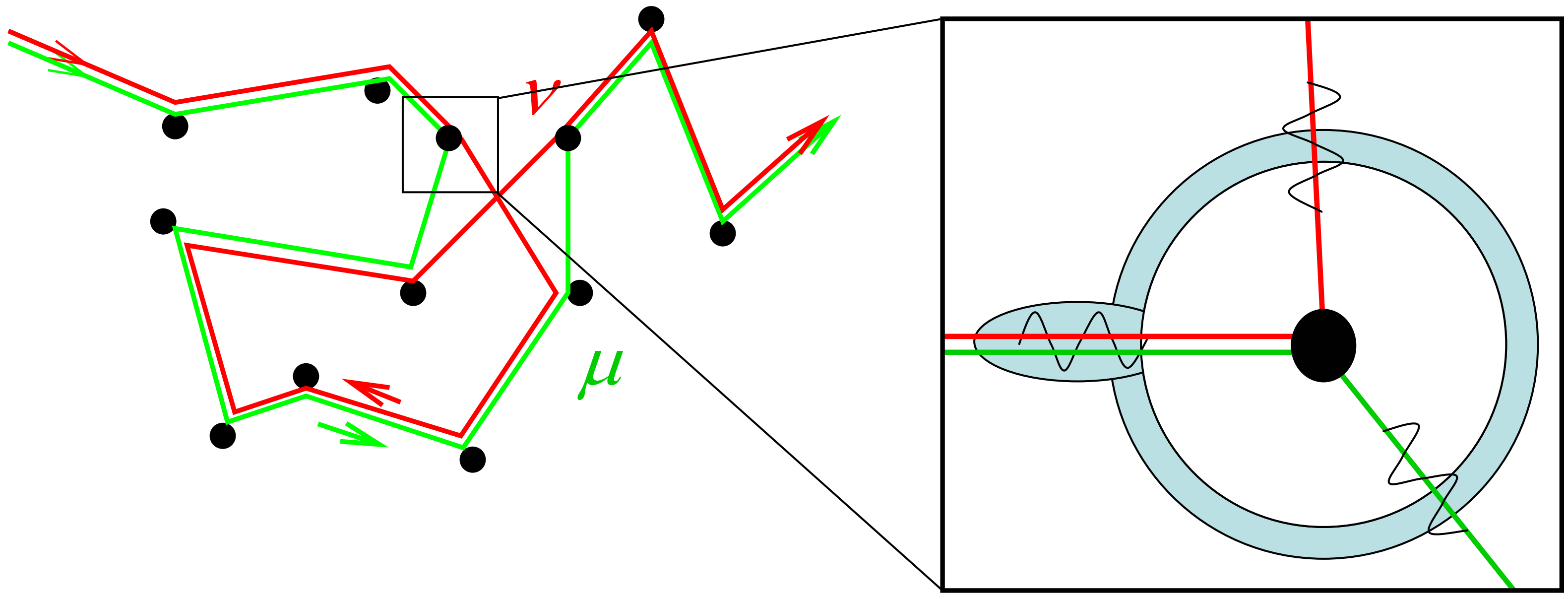


Interfering trajectories
for disordered metal

Anderson, Abrahams, Ramakrishnan (1979)
Gorkov, Larkin, Khmelnitskii (1979)

Detour: Weak localization

Negative correction $\langle \delta G \rangle$ to ensemble averaged conductance
Origin: quantum interference



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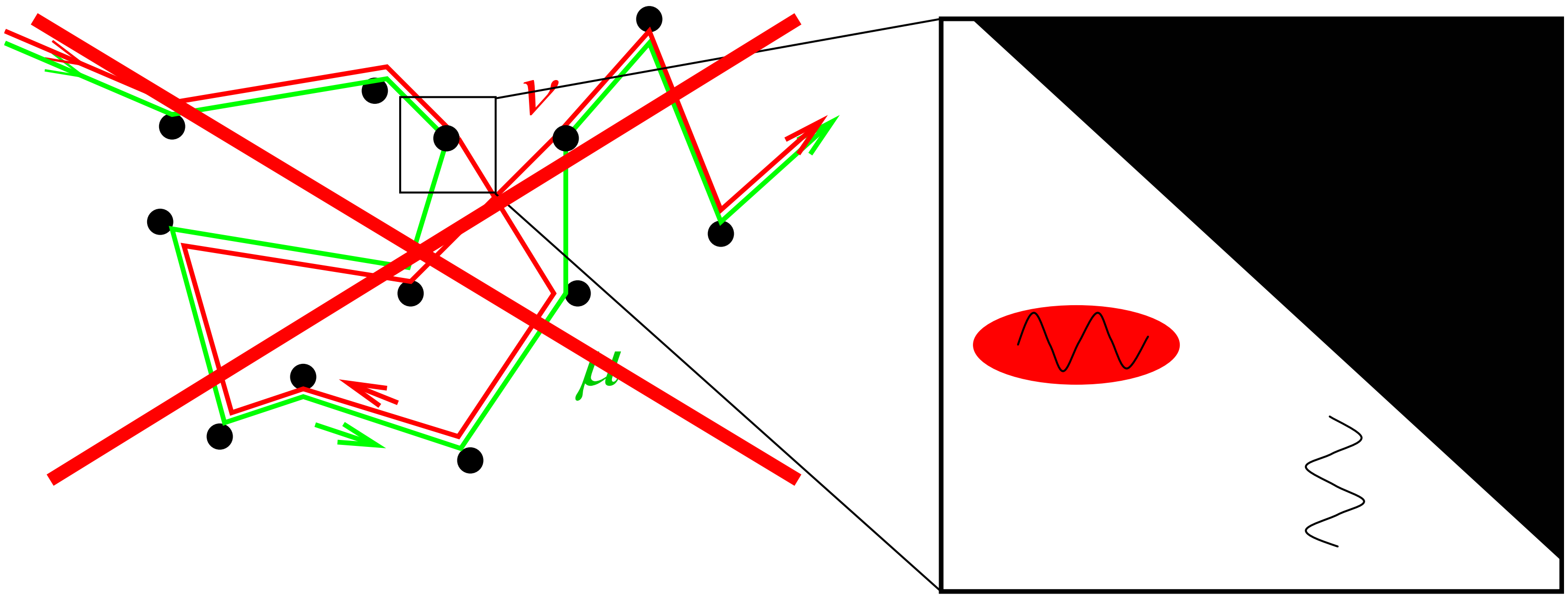
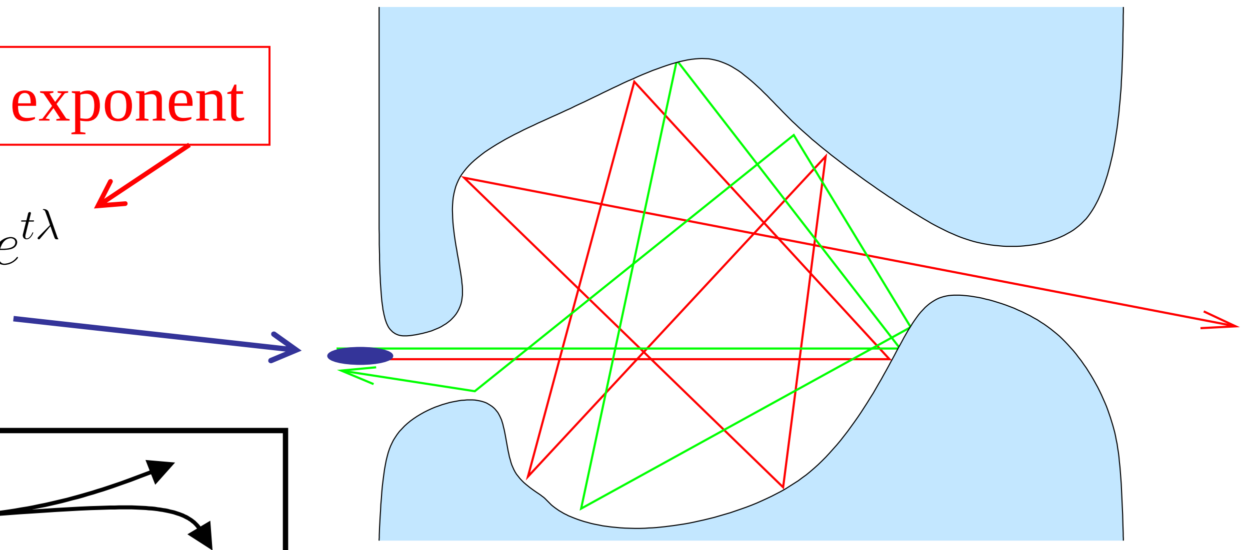


Ballistic quantum dots

λ: Lyapunov exponent

$$\delta\mathbf{r}(t) = \delta\mathbf{r}(0)e^{t\lambda}$$



separation α $e^{\lambda t}$

Quantum uncertainty in position or direction of incoming wavepacket is magnified by chaotic boundary scattering.

Aleiner and Larkin (1996)

Ehrenfest time

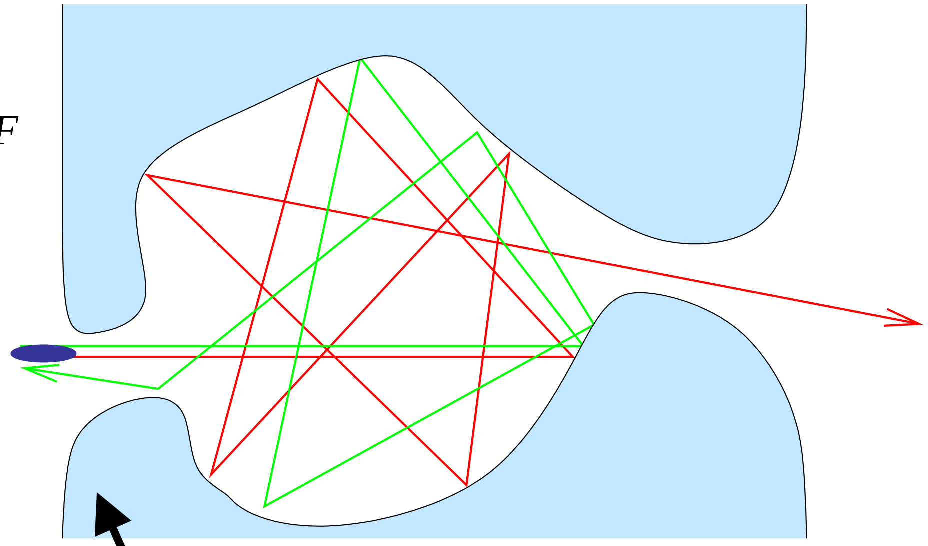
`Ehrenfest time' τ_E: Time until initial uncertainty $1/k_F$ has reached dot size L:

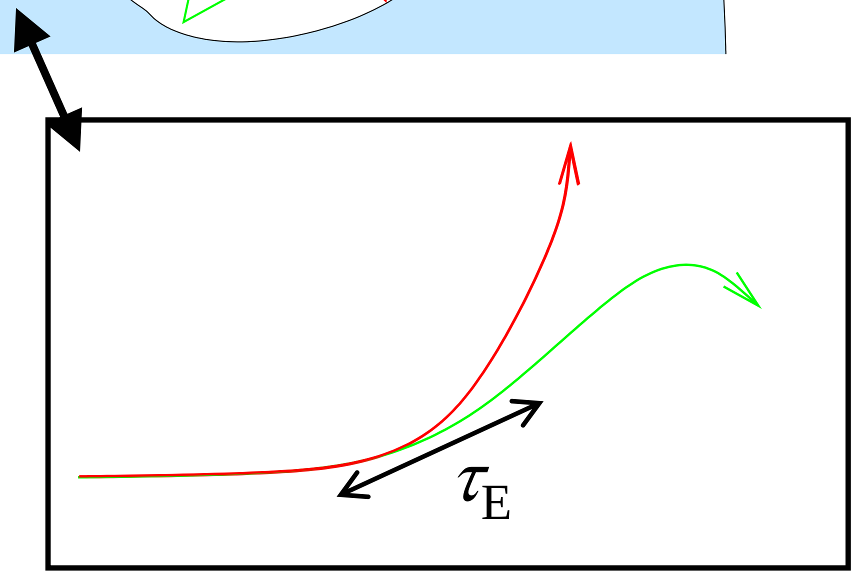
$$L = \frac{1}{k_F} e^{\lambda \tau_E}$$

$$\tau_E = \frac{1}{\lambda} \ln k_F L$$

λ: Lyapunov exponent

Random Matrix Theory valid if $\tau_E << \tau_D$





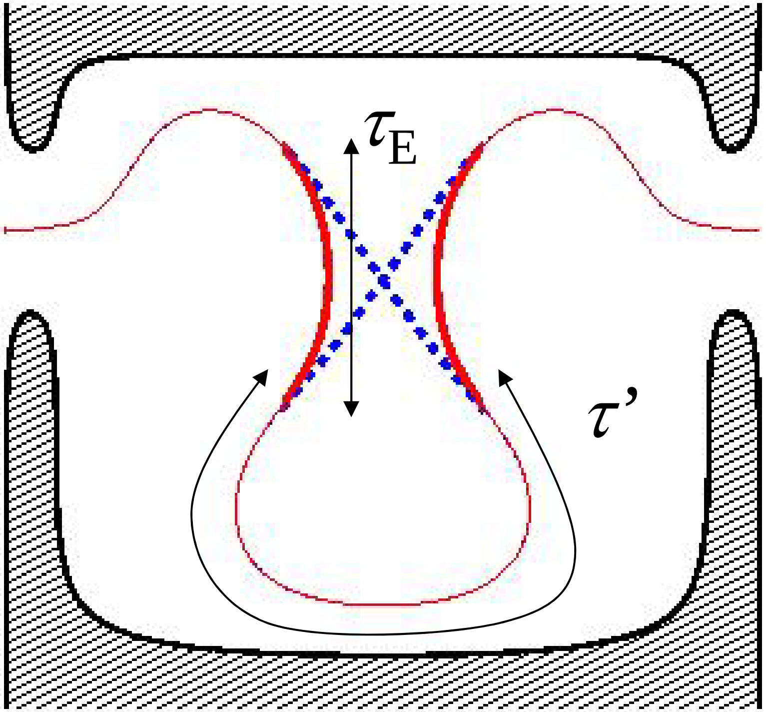
Weak localization

Probability to remain in dot:

$$P_{\text{stay}} = e^{-\tau_{\text{E}}/\tau_{\text{D}}} \times e^{-\tau'/\tau_{\text{D}}}$$

special for ballistic dot; not in RMT

also for dot with point scatterers: included in RMT



Aleiner and Larkin (1996)

$$\delta G = \delta G_{\text{RMT}} e^{-\tau_{\text{E}}/\tau_{\text{D}}}$$

$$= \delta G_{\text{RMT}} \, (k_F L)^{-1/\lambda\tau_{\text{D}}}$$

$$\tau_{\text{E}} = \frac{1}{\lambda} \ln k_F L$$

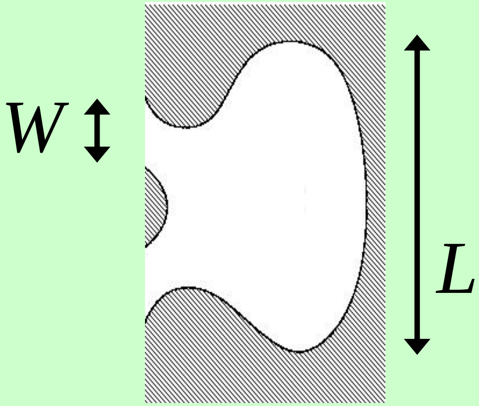
Rahav and PWB (2005)

Quantum Transport

Random matrix theory,
Disordered quantum dot

$$\delta G_{\mathrm{RMT}} = \frac{1}{4}\left(\frac{2e^2}{h}\right)$$

$$\mathrm{var}\, G_{\mathrm{RMT}} = \frac{1}{16}\left(\frac{2e^2}{h}\right)$$



Jalabert, Pichard, Beenakker (1994)
Baranger and Mello (1994)

Quantum Transport

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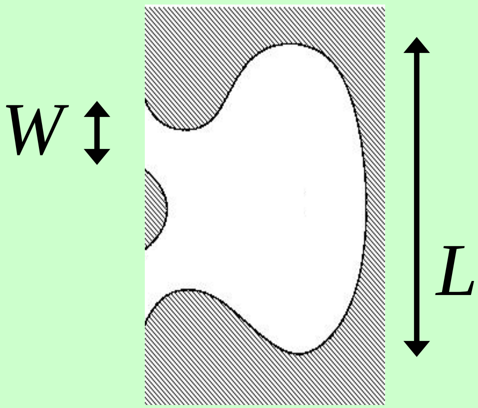

Aleiner and Larkin (1996)
Adagideli (2003)
Tworzydlo, Tajic, Beenakker (2004)
Jacquod and Sukhorukov (2004)
PWB and Rahav (2005, 2006)

Ballistic quantum dot

$$\delta G = \delta G_{\mathrm{RMT}}\, (k_F L)^{-1/\lambda\tau_{\mathrm{D}}}$$

$$\mathrm{var}\, G = \mathrm{var}\, G_{\mathrm{RMT}}$$

$$\frac{1}{\lambda\tau_{\mathrm{D}}} \sim \frac{W}{L}$$

Quantum Transport

Random matrix theory,
Disordered quantum dot

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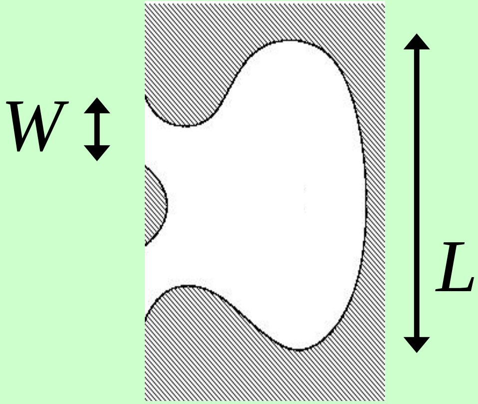


Ballistic quantum dot

$$\delta G = \delta G_{\mathrm{RMT}}\,(k_F L)^{-1/\lambda\tau_{\mathrm{D}}}$$

$$\mathrm{var}\, G = \mathrm{var}\, G_{\mathrm{RMT}}$$

Quantum Pump ?

Semiclassical Theory,
'diagonal approximation'

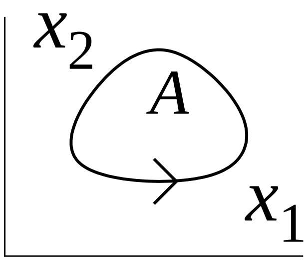
$$\langle Q^2 \rangle = \langle Q^2 \rangle_{\mathrm{RMT}}$$

Martinez-Marez, Mucciolo,
Lewenkopf (2004)

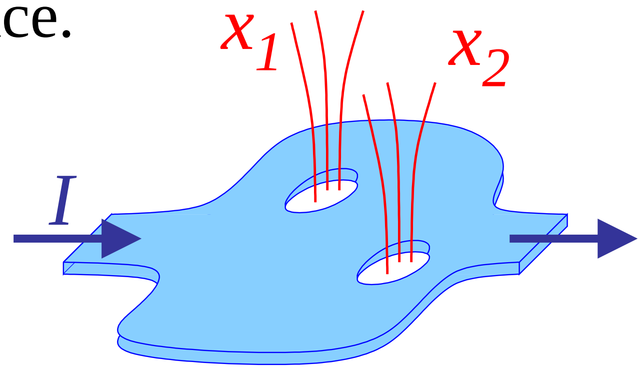
Adiabatic quantum pump

Charge pumped in one cycle:

$$Q = \frac{e}{\pi}\int_A dx_1 dx_2 \sum_n \mathrm{Im}\, \frac{\partial S_{mn}}{\partial x_2}\frac{\partial S^*_{mn}}{\partial x_1}.$$



- Q is sample specific;
  depends on quantum interference.



$\langle Q \rangle = 0$, by symmetry

Need semiclassical theory to calculate $\langle Q^2 \rangle$

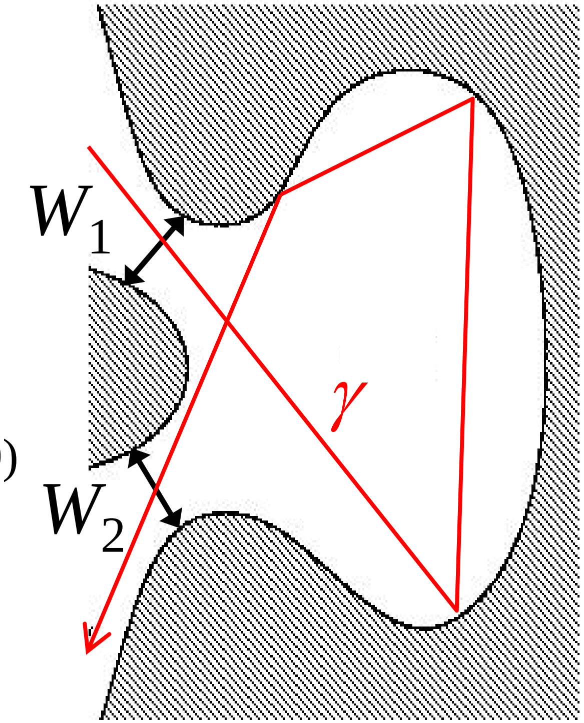
Semiclassical theory

Charge pumped in one cycle:

$$Q = \frac{e}{\pi}\int_A dx_1 dx_2 \sum_n \mathrm{Im}\,\frac{\partial S_{mn}}{\partial x_2}\frac{\partial S^*_{mn}}{\partial x_1}.$$

$$S_{mn} \sim \sum_\gamma A_\gamma e^{iS_\gamma}$$

Jalabert, Baranger, Stone (1990)



- S_γ: classical action
- angle of γ consistent with transverse momentum in lead,

$$p_\perp(m) = \pm\pi\hbar m/W_j, \quad m = 1, \ldots, N_j,$$

- A_γ: stability amplitude

Semiclassical theory

Charge pumped in one cycle:

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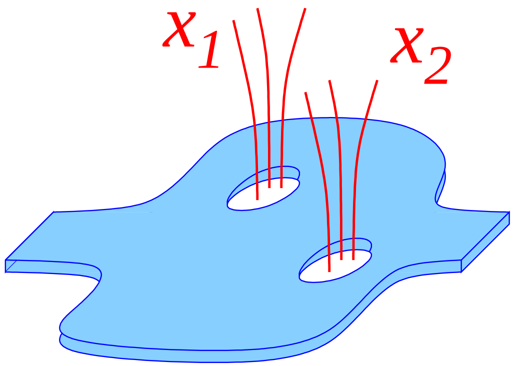

- $x_1 = x_{10}\cos(\omega t + \theta_1)$
- $x_2 = x_{20}\cos(\omega t + \theta_2)$

- S_γ: classical action

Parameters x_1, x_2 affect action S_γ of trajectory γ:

$$\langle \delta\mathcal{S}_{\gamma j}(t)\rangle = 0 \quad j = 1, 2$$

$$\langle \delta\mathcal{S}_{\gamma i}(t)\delta\mathcal{S}_{\gamma j}(t')\rangle = \hbar^2 t_\gamma C_j \delta_{ij}\cos(\omega t + \theta_j)\cos(\omega t' + \theta_j)$$

Semiclassical theory

Charge pumped in one cycle:

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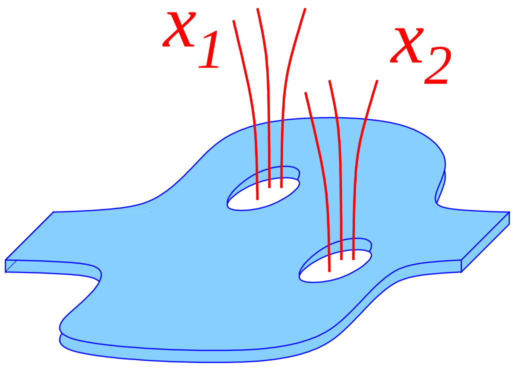


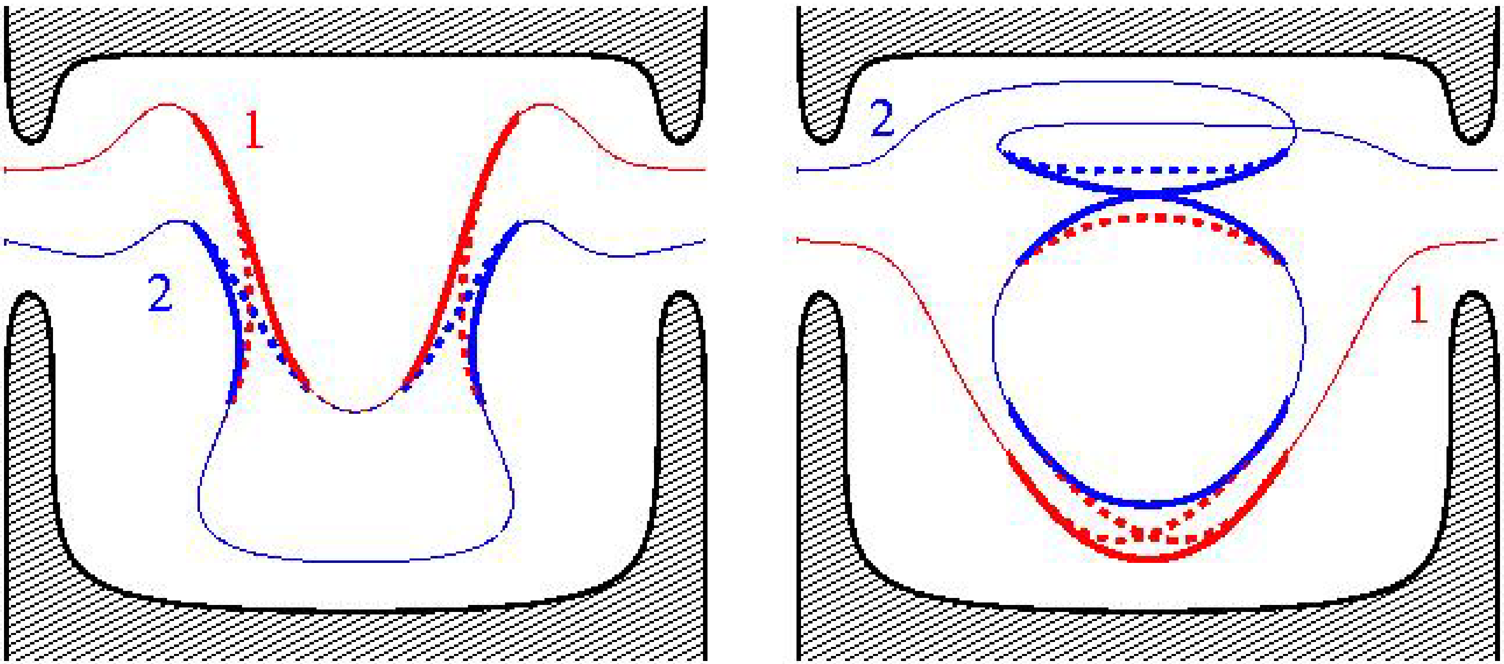
- $x_1 = x_{10} \cos(\omega t + \theta_1)$
- $x_2 = x_{20} \cos(\omega t + \theta_2)$

- S_γ: classical action

Semiclassical theory for $\langle Q^2 \rangle$: Fourfold sum over trajectories

Semiclassical Theory

Two classes of trajectories contributing to $\langle Q^2 \rangle$:



Semiclassical Theory

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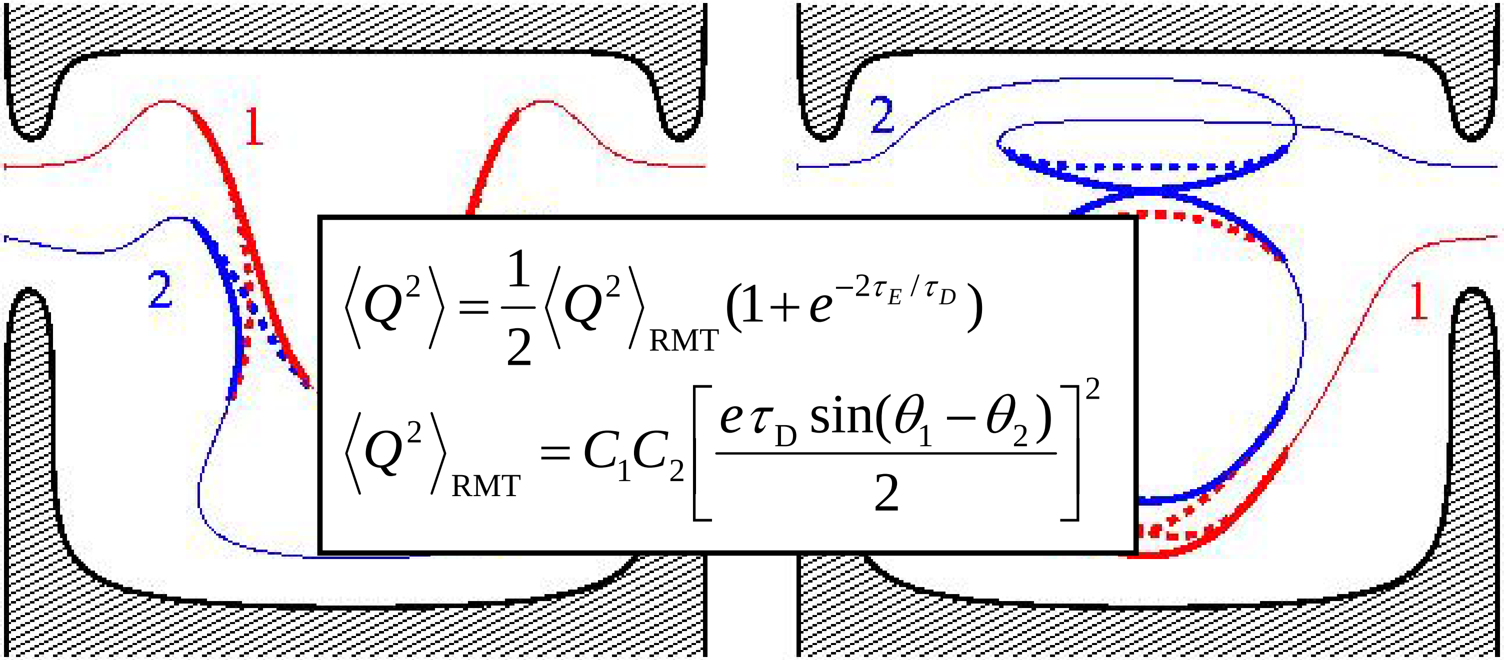


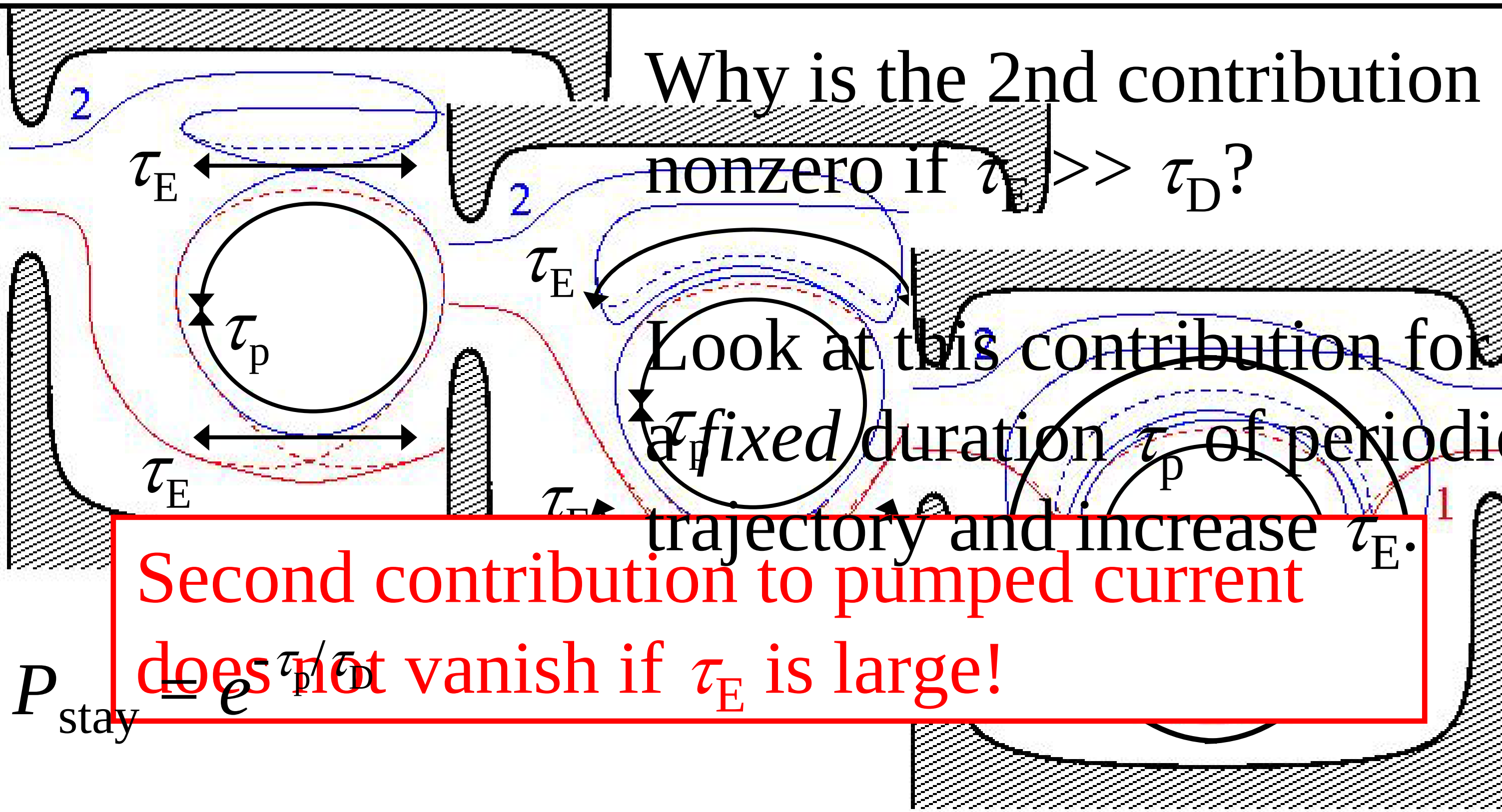
$$\left\langle Q^2 \right\rangle = \frac{1}{2}\left\langle Q^2 \right\rangle_{\mathrm{RMT}} (1 + e^{-2\tau_E/\tau_D})$$

$$\left\langle Q^2 \right\rangle_{\mathrm{RMT}} = C_1 C_2 \left[\frac{e\tau_{\mathrm{D}} \sin(\theta_1 - \theta_2)}{2} \right]^2$$

$$\left\langle Q^2 \right\rangle = \left\langle Q^2 \right\rangle_{\mathrm{RMT}} e^{-2\tau_E/\tau_D}$$

$$\left\langle Q^2 \right\rangle = \frac{1}{2}\left\langle Q^2 \right\rangle_{\mathrm{RMT}} (1 - e^{-2\tau_E/\tau_D})$$

Semiclassical Theory



Why is the 2nd contribution nonzero if $\tau_E >> \tau_D$?

Look at this contribution for a *fixed* duration τ_p of periodic trajectory and increase τ_E.

Second contribution to pumped current does not vanish if τ_E is large!

$P_{stay} = e^{-\tau_p/\tau_D}$

Duration of trajectories depends on τ_E, but P_{stay} depends on τ_p only, not on τ_E

Numerical simulations

$$\tau_{\mathrm{E}} = \frac{1}{\lambda} \ln k_F L \sim \frac{1}{\lambda} \ln N$$

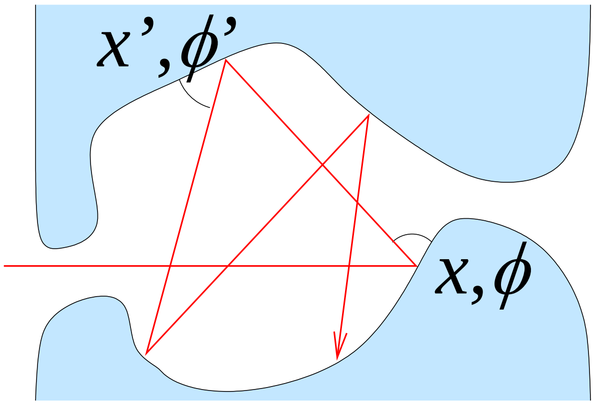
$N \sim k_F W$: #channels in point contact

Large τ_{E} ➞ exponentially large N

Trick: simulate `map’ instead of cavity.

$(x,\phi) \rightarrow (x',\phi')$

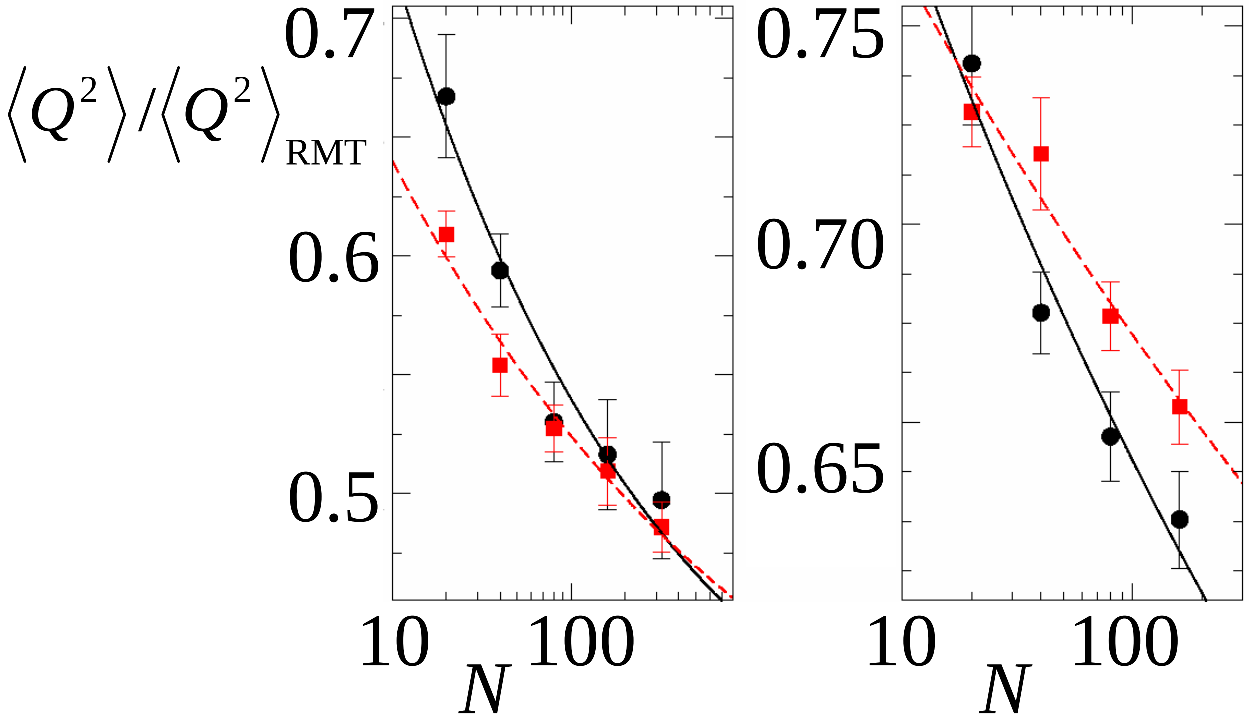
Convenient map:
Quantum kicked rotator



Three kick model, with two parameters.

Jacquod, Schomerus, Beenakker (2003)
Tworzydlo, Tajic, Beenakker (2004)

Numerical simulations

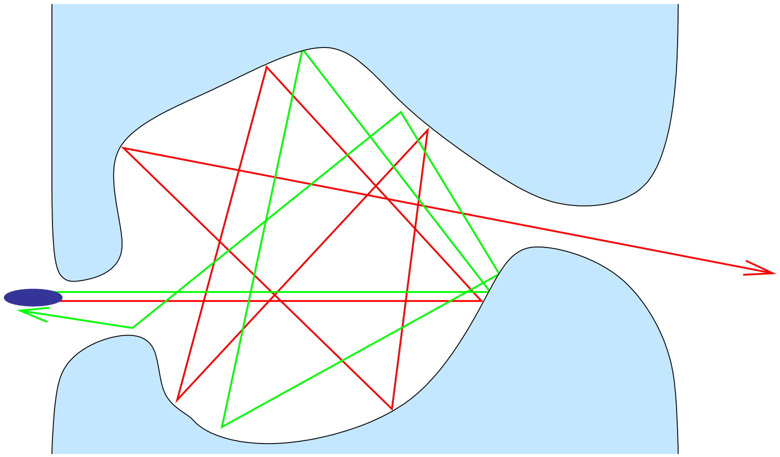


Simulations consistent with $\langle Q^2 \rangle / \langle Q^2 \rangle_{\mathrm{RMT}} = a + be^{-\tau_E/\tau_D}$,
but a, b differ from theoretical value a, b = ½.

$\tau_{\mathrm{D}} >> \tau_{\mathrm{erg}}$ not obeyed in simulations

Conclusion

Wave phenomena in ballistic quantum dots only appear after the Ehrenfest time τ_{E}.



weak localization	$\delta G = \delta G_{\mathrm{RMT}} \times e^{-\tau_{\mathrm{E}}/\tau_{\mathrm{D}}}$
conductance fluctuations	$\mathrm{var}\, G = \mathrm{var}\, G_{\mathrm{RMT}}$
shot noise power	$P = P_{\mathrm{RMT}} \times e^{-\tau_{\mathrm{E}}/\tau_{\mathrm{D}}}$ Agam, Aleiner, Larkin (2000)
quantum pump	$\langle I^2 \rangle = \langle I^2_{\mathrm{RMT}} \rangle \times \dfrac{1 + e^{-2\tau_{\mathrm{E}}/\tau_{\mathrm{D}}}}{2}$